Decomposition of Complex Reaction Networks into Building Blocks

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Introduction

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- The purpose of this study is to develop a method allowing to the study the property and the structure of any reaction network
- It is based on the method developed for the analysis of metabolic network.

Chemical Network

A system of chemical reactions

Example system:

$$X_1 + X_2 \xrightarrow{R_1} X_3$$

$$X_3 + X_4 \xrightarrow{R_2} X_5 + X_6$$

$$X_6 \xrightarrow{R_3} 2X_2$$

$$\xrightarrow{R_4} X_4$$

$$X_5 \xrightarrow{R_5}$$

$$\xrightarrow{R_6} X_1$$

$$X_2 \xrightarrow{R_7}$$

A system of chemical reactions

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$$X_3 + X_4 \xrightarrow{R_2} X_5 + X_6 \qquad X_5 \xrightarrow{R_5}$$

$$X_6 \xrightarrow{R_3} 2X_2 \qquad \xrightarrow{R_6} X_1$$

$$X_2 \xrightarrow{R_7}$$

Formal representation:

$$\forall j \in [1, r] \qquad R_j \equiv \sum_{i=1}^{n} \nu_{i,j} X_i = 0$$

The stoichiometric matrix

System described by the stoichiometric coefficients:

$$\nu = [\nu_{i,j}]_{i \in [1,n], j \in [1,r]}$$

$\nu_{i,j}$ is the stoichiometric coefficient of the compound i in the transformation j. It is by convention positive for products (formed compounds), and negative for reactants (disappearing compounds).

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$$\nu = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & +1 & 0 \\ -1 & 0 & +2 & 0 & 0 & 0 & -1 \\ +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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Right nullspace

The right nullspace is the space of all possible combination of reactions that cancel themselves. This corresponds to the possible combination of reactions able to maintain a steady state

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$$\mathrm{Null}\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & +1 & 0 \\ -1 & 0 & +2 & 0 & 0 & 0 & -1 \\ +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} = \{[1111111]\}$$

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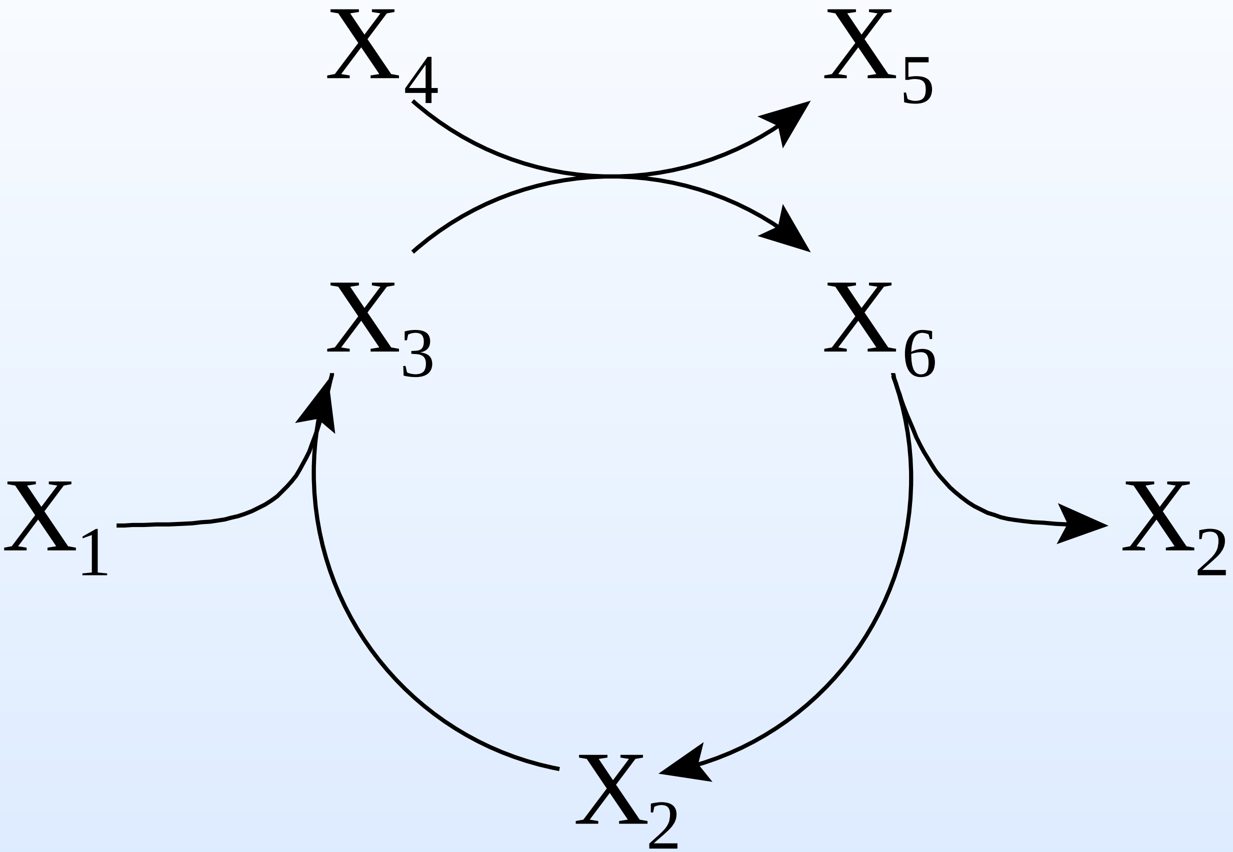
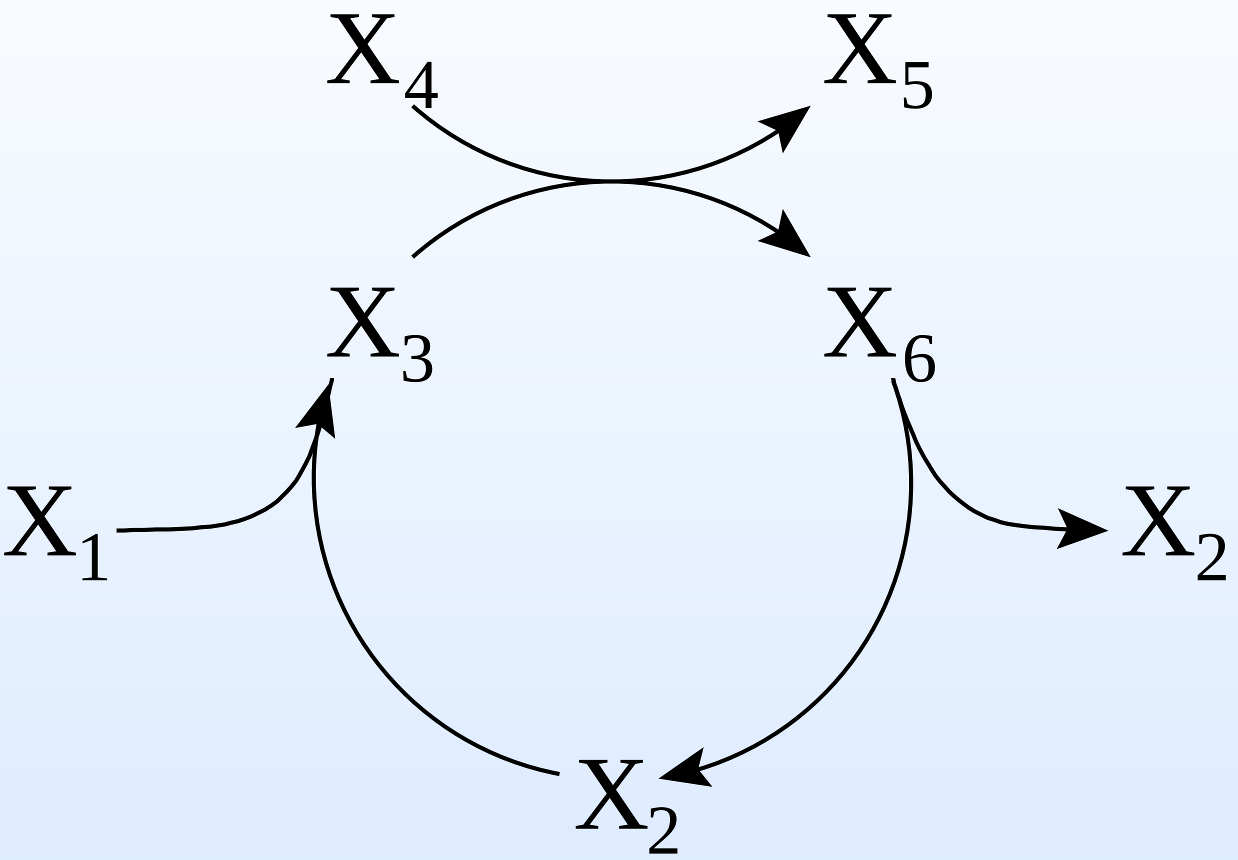
The right nullspace contains only one vector, that is the only steady state that can be established with the corresponding flows involves each reaction once.

Network Decomposition

Global mode



Global mode



Poorly informative. Further decomposition possible ?

Building blocks

A building block is a molecule subelement that is never broken during a chemical reaction of the network.

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$$S_1 + S_2 \xrightarrow{R_1} S_1 S_2$$

Building blocks

A building block is a molecule subelement that is never broken during a chemical reaction of the network.
For example:

$$X_1 + X_2 \xrightarrow{R_1} X_3$$

$$S_1 + S_2 \xrightarrow{R_1} S_1S_2$$

The mass balance imposes that there is the same number in each element on both sides of the reactions.

Left nullspace

If molecules can be decomposed by:

$$X_i = \sum_{k=1}^{s} \sigma_{i,k} S_k$$

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then the mass balance for each reaction gives:

$$\Rightarrow \forall j \in [1, r] \ , \forall k \in [1, s] \ , \quad 0 = \sum_{i=1}^{n} \nu_{i,j} \sigma_{i,k}$$

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$$\Leftrightarrow \sigma_k^t \cdot \nu = 0$$

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$$\Leftrightarrow \sigma_k^t \cdot \nu = 0$$

The computation of one base of the left nullspace – i.e. the linear combinations of rows that gives zero – gives the decomposition of molecules into constitutive building blocks.

Example decomposition

Nullspace:

$$\left\{ \begin{bmatrix} 1 & 1 & 2 & 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \right\}$$

Example decomposition

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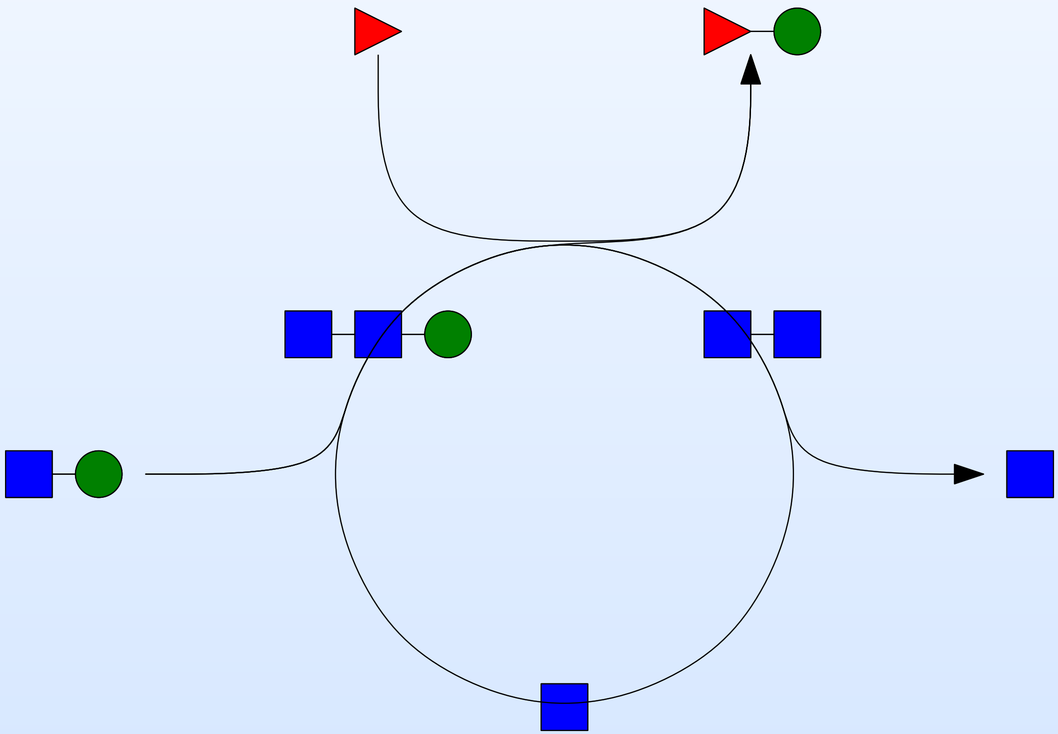

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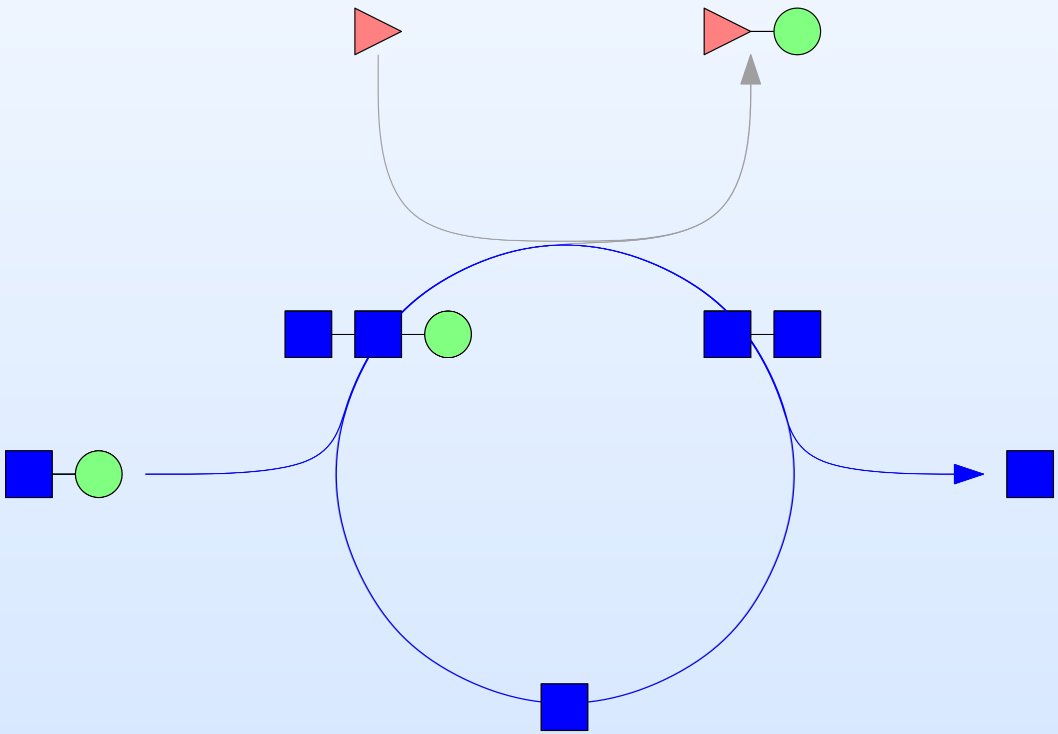

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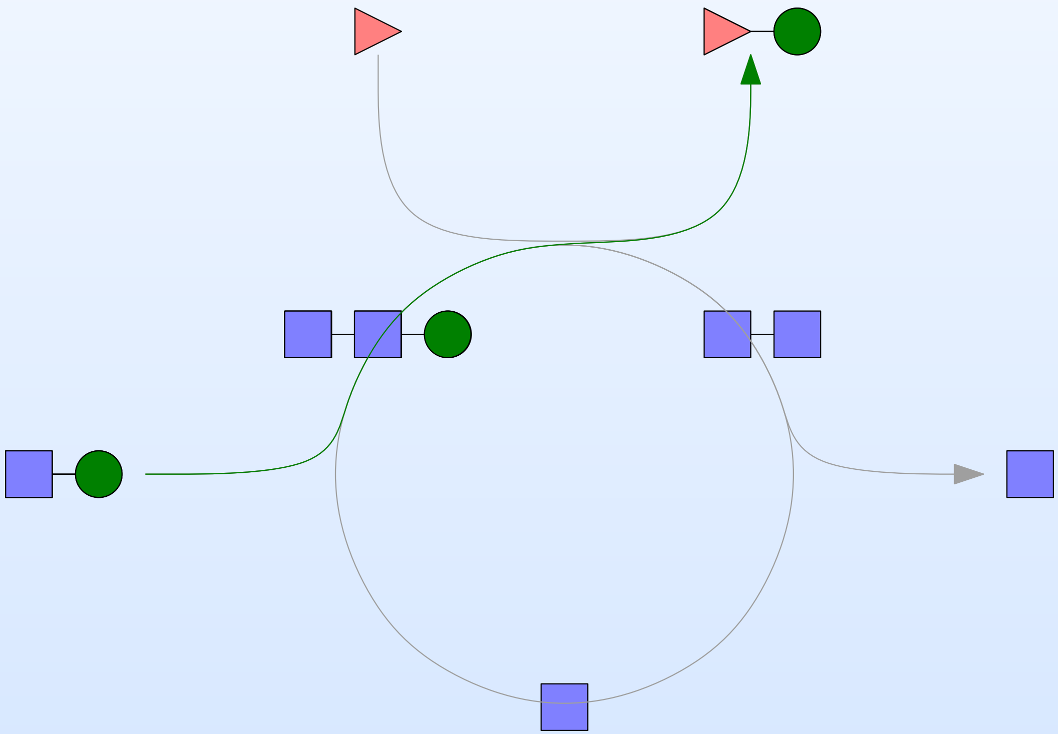

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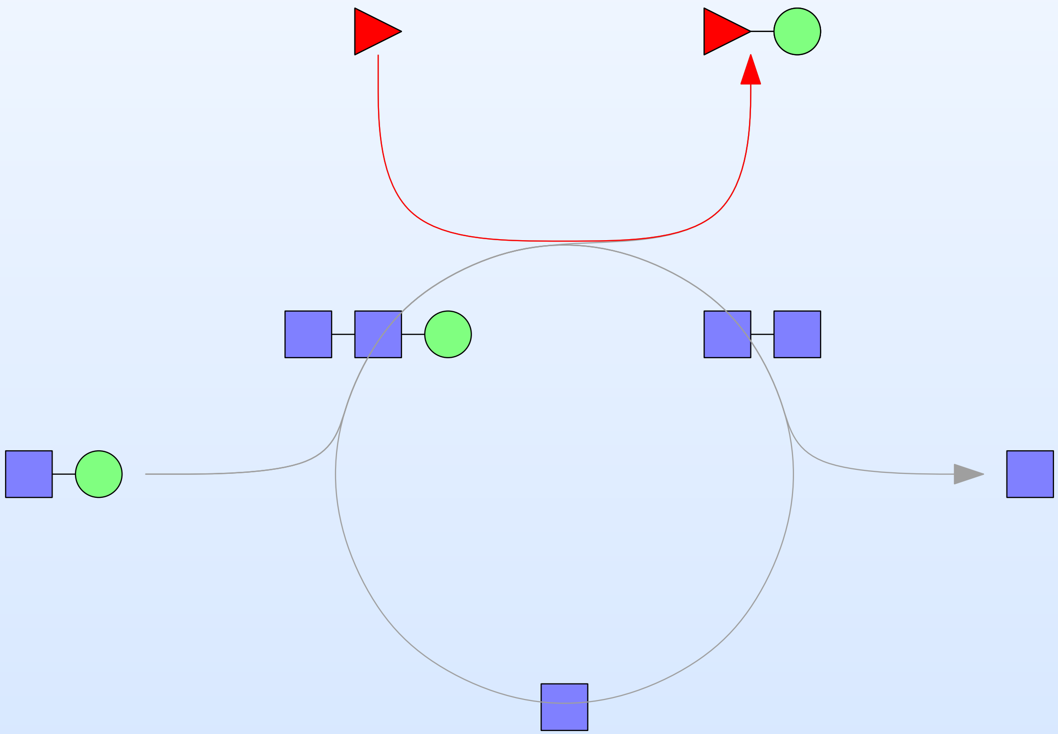
$$\left\{ \begin{bmatrix} 1 & 1 & 2 & 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \right\}$$



Computation of the decomposition

Stoichimoetric matrix of the whole system:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & +1 & 0 \\ -1 & 0 & +2 & 0 & 0 & 0 & -1 \\ +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Computation of the decomposition

Put the global flux corresponding to the nullspace:

$$\begin{bmatrix} -1 & 0 & 0 & +1 \\ -1 & 0 & +2 & -1 \\ +1 & -1 & 0 & 0 \\ 0 & -1 & 0 & +1 \\ 0 & +1 & 0 & -1 \\ 0 & +1 & -1 & 0 \end{bmatrix}$$

Computation of the decomposition

Element wise multiplication with the building block vector gives its restricted stoichiometric matrix.

$$\begin{bmatrix} -1 & 0 & 0 & +1 \\ -1 & 0 & +2 & -1 \\ +1 & -1 & 0 & 0 \\ 0 & -1 & 0 & +1 \\ 0 & +1 & 0 & -1 \\ 0 & +1 & -1 & 0 \end{bmatrix} \quad \text{for} \quad \begin{bmatrix} 1 & 1 & 2 & 0 & 0 & 2 \end{bmatrix}$$

Computation of the decomposition

Element wise multiplication with the building block vector gives its restricted stoichiometric matrix.

$$\begin{bmatrix} -1 & 0 & 0 & +1 \\ -1 & 0 & +2 & -1 \\ +2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & +2 & -2 & 0 \end{bmatrix}$$

Computation of the decomposition

Separation of the multiple molecule and reactions.

$$
\left[\begin{array}{cc|cc|cc|c}
-1 & 0 & 0 & 0 & 0 & 0 & +1 \\
\hline
0 & -1 & 0 & 0 & +1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & +1 & -1 \\
\hline
+1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & +1 & 0 & -1 & 0 & 0 & 0 \\
\hline
0 & 0 & +1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & +1 & 0 & -1 & 0
\end{array}\right]
$$

Computation of the decomposition

Separation of the multiple molecule and reactions.

	$R_1^{(3,a)}$	$R_1^{(3,b)}$	$R_2^{(3,a)}$	$R_2^{(3,b)}$	$R_3^{(3,a)}$	$R_3^{(3,b)}$	$F^{(3)}$
$X_1^{(3)}$	-1	0	0	0	0	0	$+1$
$X_2^{(3,a)}$	0	-1	0	0	$+1$	0	0
$X_2^{(3,b)}$	0	0	0	0	0	$+1$	-1
$X_3^{(3,a)}$	$+1$	0	-1	0	0	0	0
$X_3^{(3,b)}$	0	$+1$	0	-1	0	0	0
$X_6^{(3,a)}$	0	0	$+1$	0	-1	0	0
$X_6^{(3,b)}$	0	0	0	$+1$	0	-1	0

Square matrix, one and only one $+1/-1$ pair in each column and row.

Computation of the decomposition

Simplification of the obtained cycles:

$$
\left[\begin{array}{cc|cc|cc|c}
-1 & \leftarrow & \leftarrow & \leftarrow & \leftarrow & \leftarrow & +1 \\
\hline
\downarrow & -1 & \leftarrow & \leftarrow & +1 & 0 & \uparrow \\
\downarrow & \downarrow & 0 & 0 & \uparrow & +1 & -1 \\
\hline
+1 & \downarrow\!\!\!\!\rightarrow & -1 & 0 & \uparrow & \uparrow & 0 \\
0 & +1 & \downarrow\!\!\!\!\rightarrow & -1 & \uparrow & \uparrow & 0 \\
\hline
0 & 0 & +1 & \downarrow\!\!\!\!\rightarrow & -1 & \uparrow & 0 \\
0 & 0 & 0 & +1 & \rightarrow & -1 & 0
\end{array}\right]
$$

Computation of the decomposition

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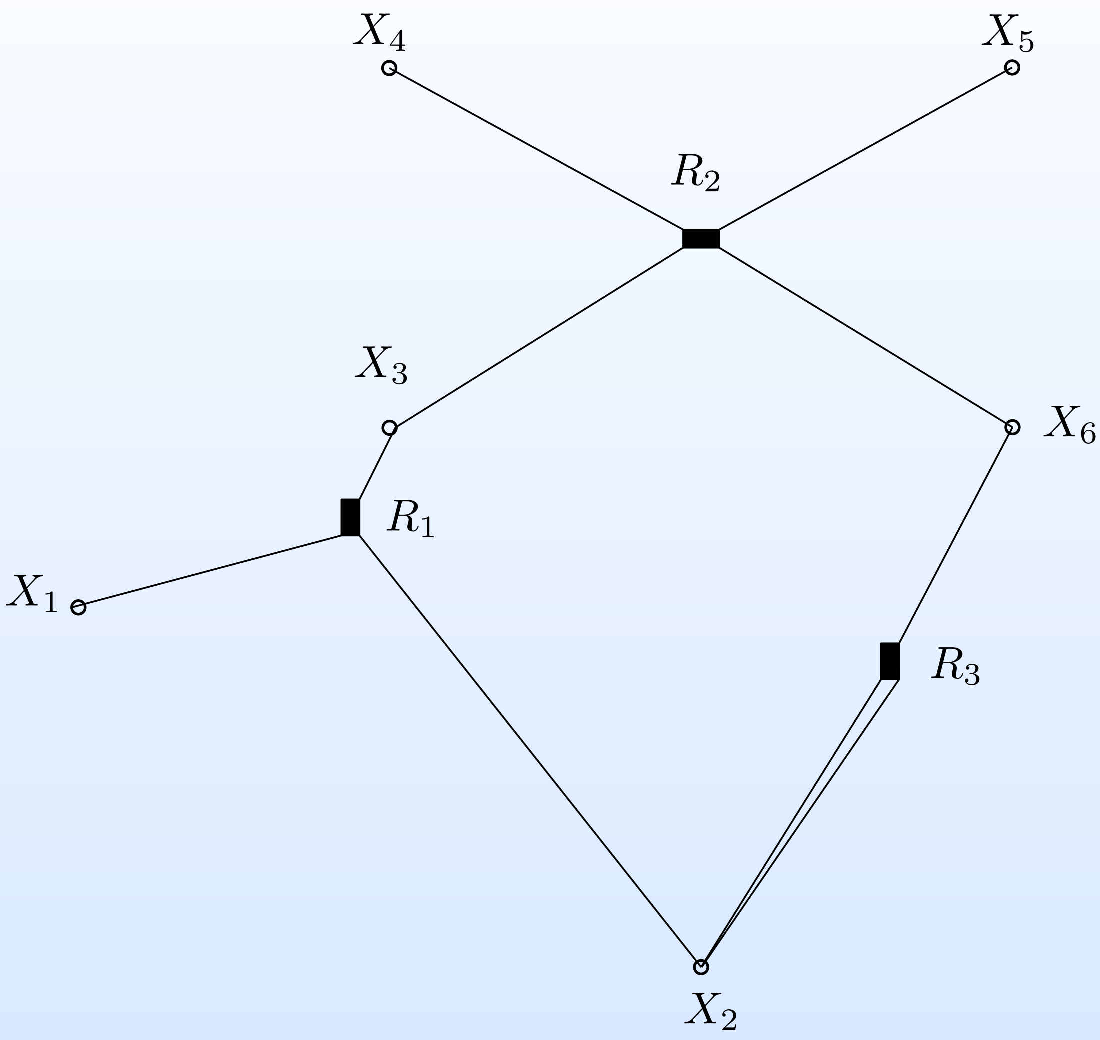
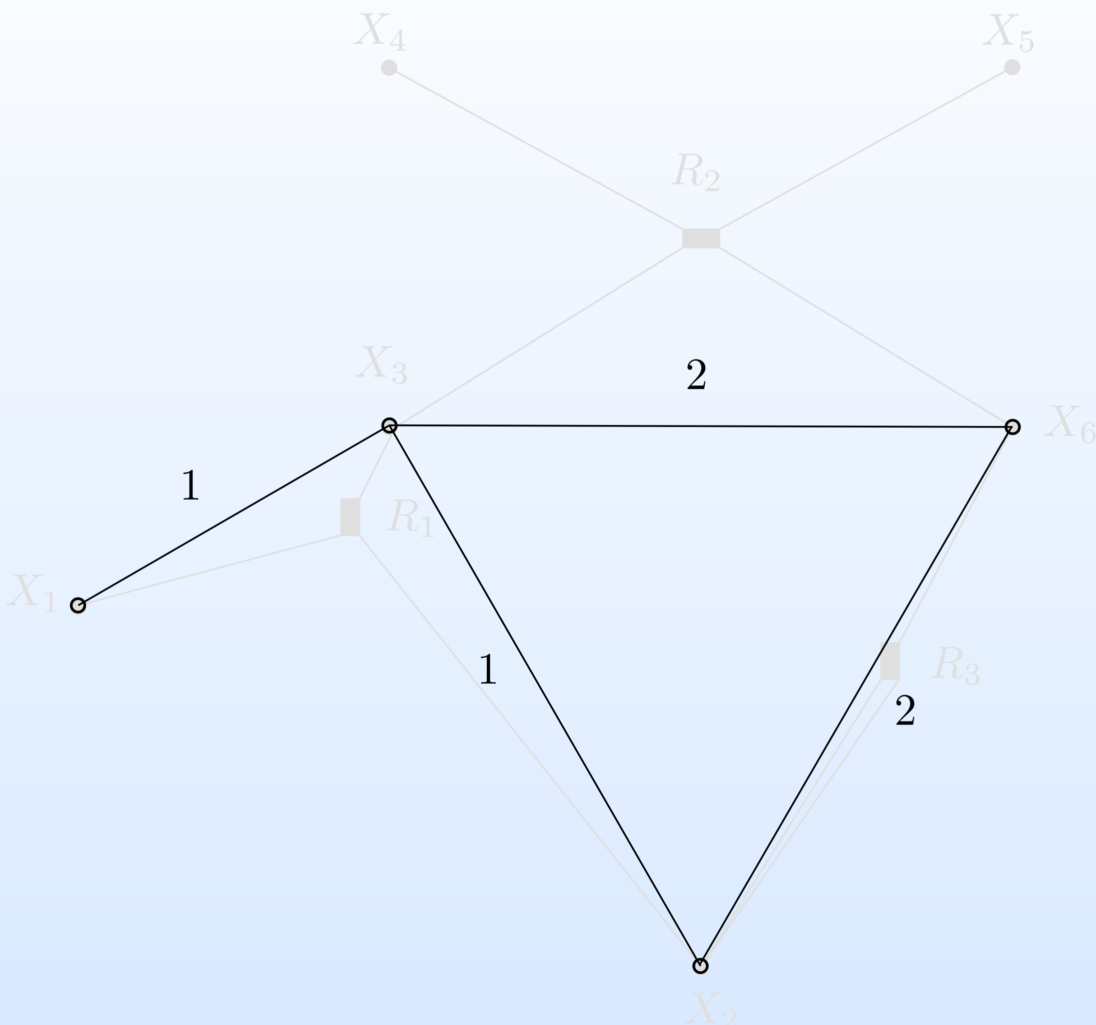
$$
\left[\begin{array}{cc|cc|cc|c}
-1 & \leftarrow & \leftarrow & \leftarrow & \leftarrow & \leftarrow & +1 \\
\hline
\downarrow & -1 & \leftarrow & \leftarrow & +1 & 0 & \uparrow \\
\downarrow & \downarrow & 0 & 0 & \uparrow & +1 & -1 \\
\hline
\downarrow & +1 & -1 & 0 & \uparrow & \uparrow & 0 \\
+1 & \rightarrow & \downarrow\!\!\!\rightarrow & -1 & \uparrow & \uparrow & 0 \\
\hline
0 & 0 & +1 & \downarrow\!\!\!\rightarrow & -1 & \uparrow & 0 \\
0 & 0 & 0 & +1 & \rightarrow & -1 & 0
\end{array}\right]
$$

Computation of the decomposition



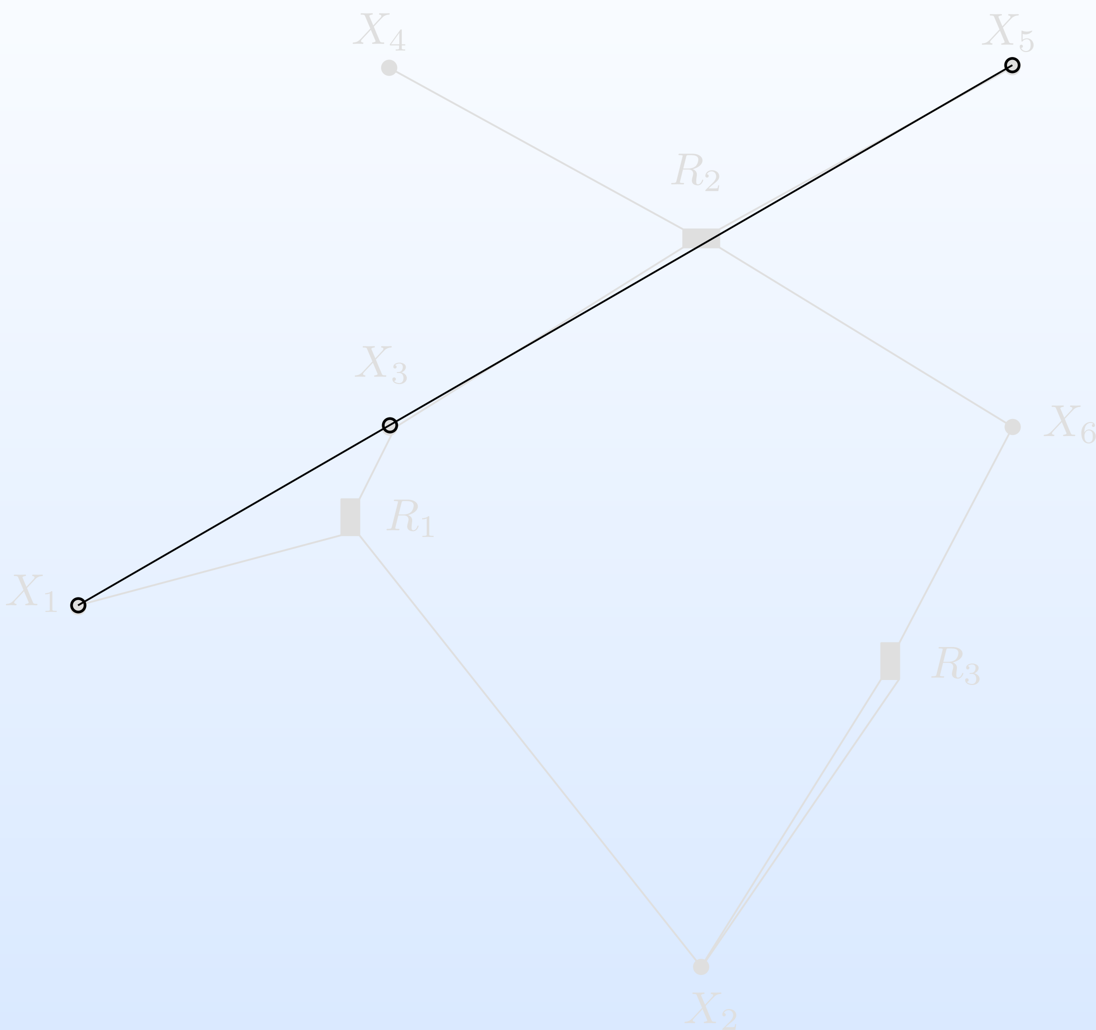
Complete network, two kind of node necessary.

Computation of the decomposition



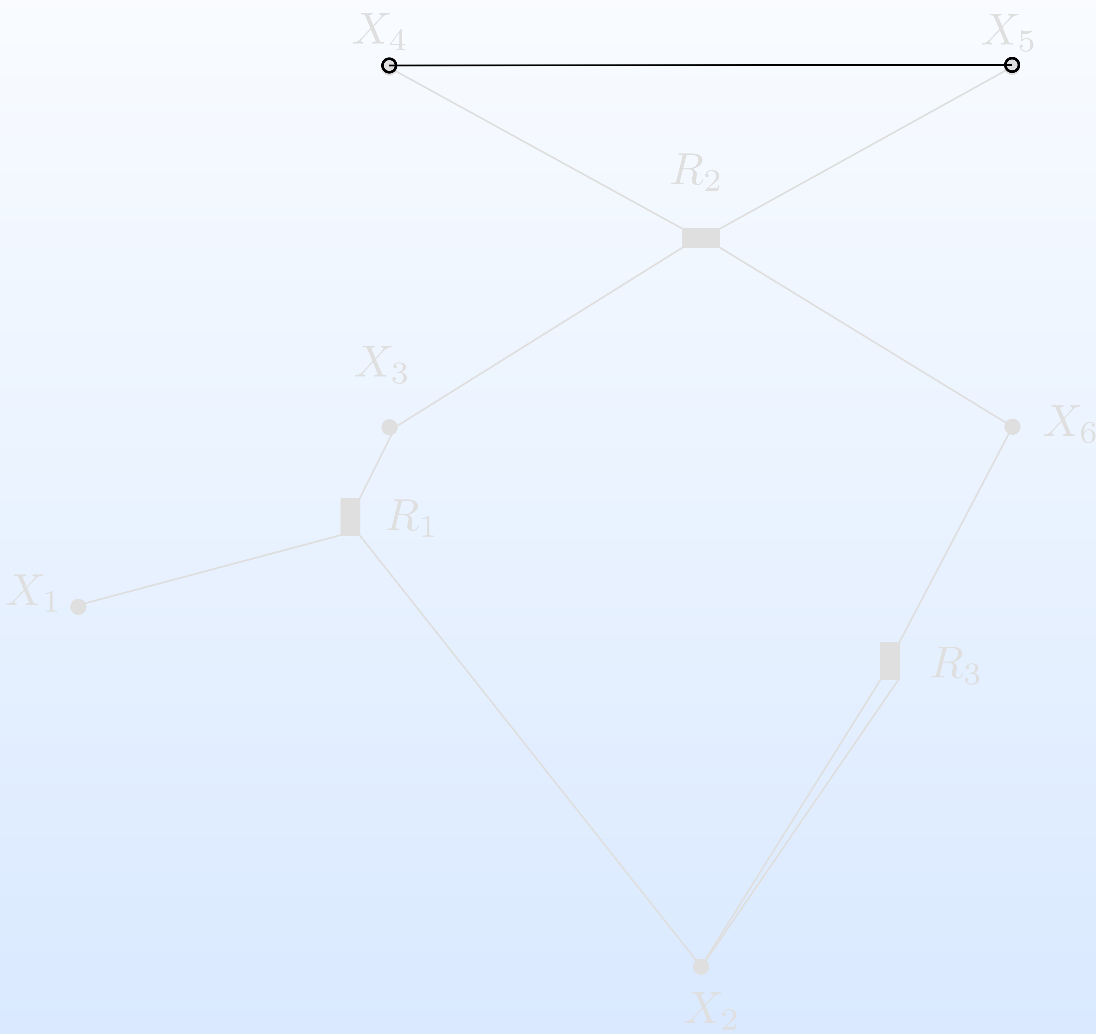
Subnetwork of first building blocks. Node disambiguation.

Computation of the decomposition



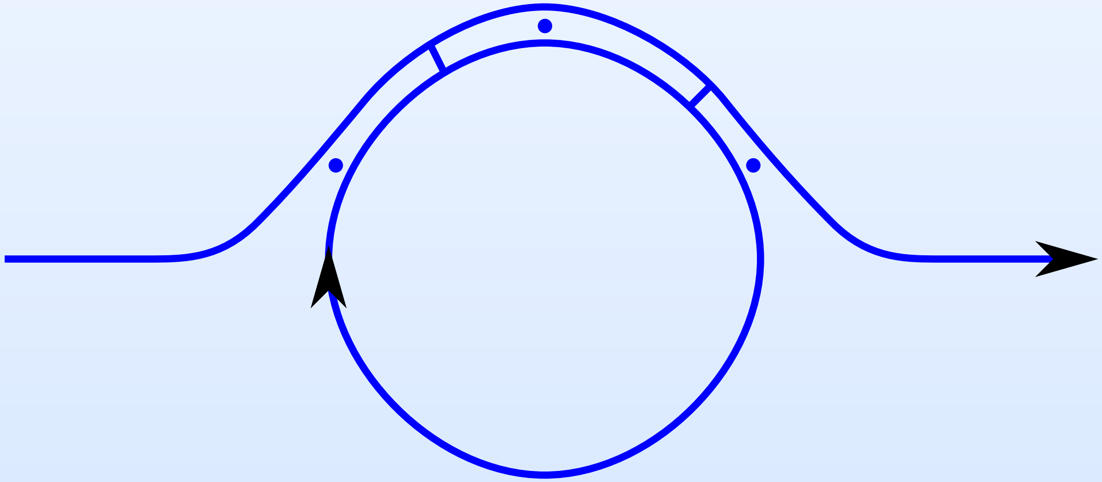
Subnetwork of second building blocks.

Computation of the decomposition



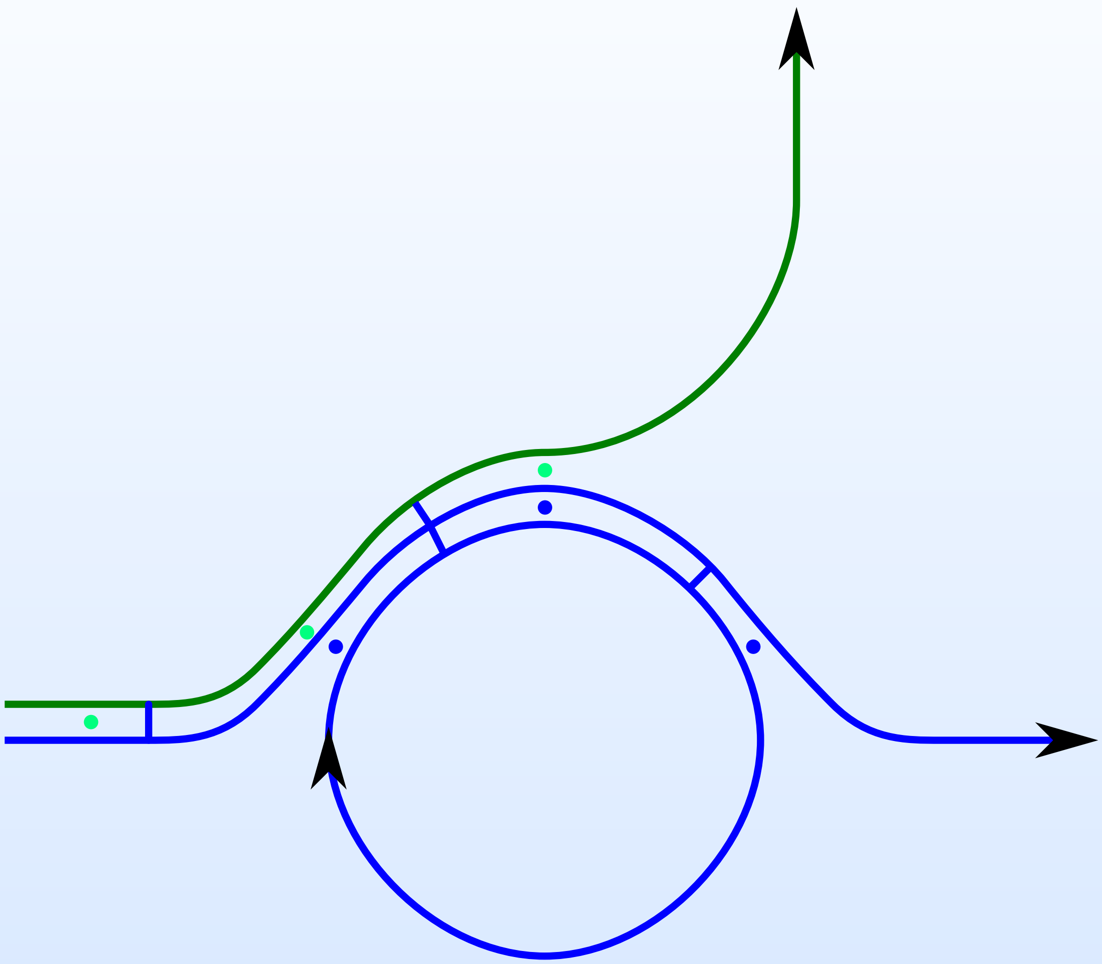
Subnetwork of third building blocks.

Computation of the decomposition



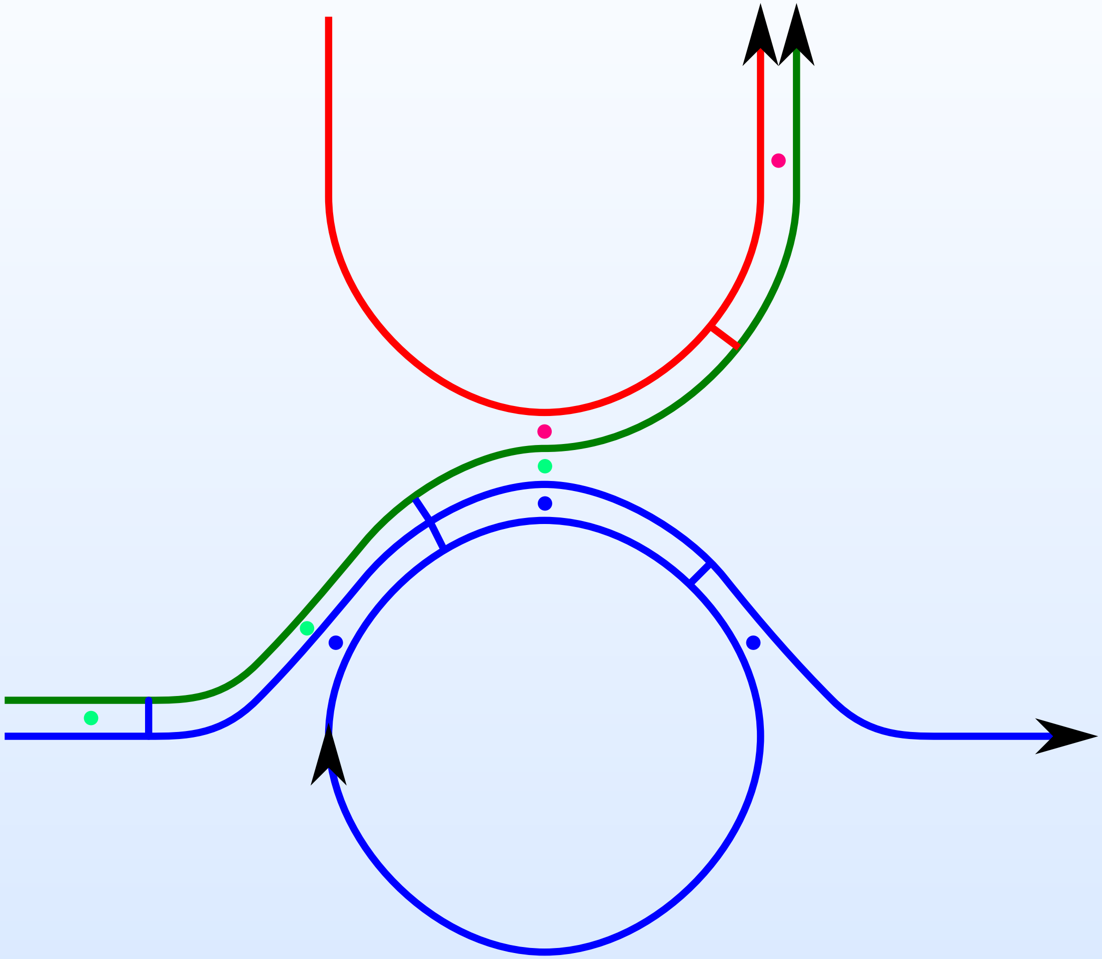
Autocatalysis.

Computation of the decomposition



Element transfer.

Computation of the decomposition



Feeding flux.

Complex systems

Partial EColi metabolism

$$
\begin{aligned}
X_1 &= CO2 \\
X_2 &= O2 \\
X_3 &= GLC \\
X_4 &= ATP \\
X_5 &= G6P \\
X_6 &= ADP \\
X_7 &= F6P \\
X_8 &= FDP \\
X_9 &= T3P1 \\
X_{10} &= T3P2 \\
X_{11} &= Pi \\
X_{12} &= NAD \\
X_{13} &= NADH \\
X_{14} &= 13PDG \\
X_{15} &= 3PG \\
X_{16} &= 2PG \\
X_{17} &= PEP \\
X_{18} &= PYR
\end{aligned}
$$

$$
\begin{aligned}
X_{20} &= COA \\
X_{21} &= ACCOA \\
X_{22} &= OA \\
X_{23} &= CIT \\
X_{24} &= ICIT \\
X_{25} &= NADP \\
X_{26} &= NADPH \\
X_{27} &= AKG \\
X_{28} &= SUCCoA \\
X_{29} &= SUCC \\
X_{30} &= FAD \\
X_{31} &= FADH \\
X_{32} &= FUM \\
X_{33} &= MAL \\
X_{34} &= Q \\
X_{35} &= QH_2 \\
X_{36} &= H \\
X_{37} &= Hext
\end{aligned}
$$

$$
\begin{aligned}
X_3 + X_4 &\xrightarrow{R_1} X_5 + X_6 \\
X_5 &\xrightarrow{R_2} X_7 \\
X_4 + X_7 &\xrightarrow{R_3} X_6 + X_8 \\
X_8 &\xrightarrow{R_4} X_9 + X_{10} \\
X_9 &\xrightarrow{R_5} X_{10} \\
X_9 + X_{11} + X_{12} &\xrightarrow{R_6} X_{13} + X_{14} \\
X_6 + X_{14} &\xrightarrow{R_7} X_4 + X_{15} \\
X_{15} &\xrightarrow{R_8} X_{16} \\
X_{16} &\xrightarrow{R_9} X_{17} \\
X_6 + X_{17} &\xrightarrow{R_{10}} X_4 + X_{18} \\
X_{12} + X_{18} + X_{20} &\xrightarrow{R_{11}} X_1 + X_{13} + X_{21} \\
X_{21} + X_{22} &\xrightarrow{R_{12}} X_{20} + X_{23}
\end{aligned}
$$

$$
\begin{aligned}
X_{23} &\xrightarrow{R_{13}} X_{24} \\
X_{24} + X_{25} &\xrightarrow{R_{14}} X_1 + X_{26} + X_{27} \\
X_{12} + X_{20} + X_{27} &\xrightarrow{R_{15}} X_1 + X_{13} + X_{28} \\
X_6 + X_{11} + X_{28} &\xrightarrow{R_{16}} X_4 + X_{20} + X_{29} \\
X_{29} + X_{30} &\xrightarrow{R_{17}} X_{31} + X_{32} \\
X_{32} &\xrightarrow{R_{18}} X_{33} \\
X_{12} + X_{33} &\xrightarrow{R_{19}} X_{13} + X_{22} \\
2.X_{13} + 2.X_{34} + 7.X_{37} &\xrightarrow{R_{20}} 2.X_{12} + 2.X_{35} + 7.X_{36} \\
X_2 + 2.X_{35} + 5.X_{37} &\xrightarrow{R_{21}} 2.X_{34} + 5.X_{36} \\
X_{31} + X_{34} &\xrightarrow{R_{22}} X_{30} + X_{35} \\
X_{13} + X_{25} + 2.X_{36} &\xrightarrow{R_{23}} X_{12} + X_{26} + 2.X_{37} \\
X_4 + 4.X_{37} &\xrightarrow{R_{24}} X_6 + X_{11} + 4.X_{36}
\end{aligned}
$$

Partial EColi metabolism

$$
\overleftrightarrow{\nu} = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&+1&0&0&+1&+1&0&0&0&0&0&0&0&0&0&-6&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&+6&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&0\\
-1&0&-1&0&0&0&+1&0&0&+1&0&0&0&0&0&+1&0&0&0&0&0&0&0&-1&0&-1\\
+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
+1&0&+1&0&0&0&-1&0&0&-1&0&0&0&0&0&-1&0&0&0&0&0&0&0&+1&0&+1\\
0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&+1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&+1&+1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&+1&0&+1\\
0&0&0&0&0&-1&0&0&0&0&-1&0&0&0&-1&0&0&0&-1&+2&0&0&+1&0&0&0\\
0&0&0&0&0&+1&0&0&0&0&+1&0&0&0&+1&0&0&0&+1&-2&0&0&-1&0&0&0\\
0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
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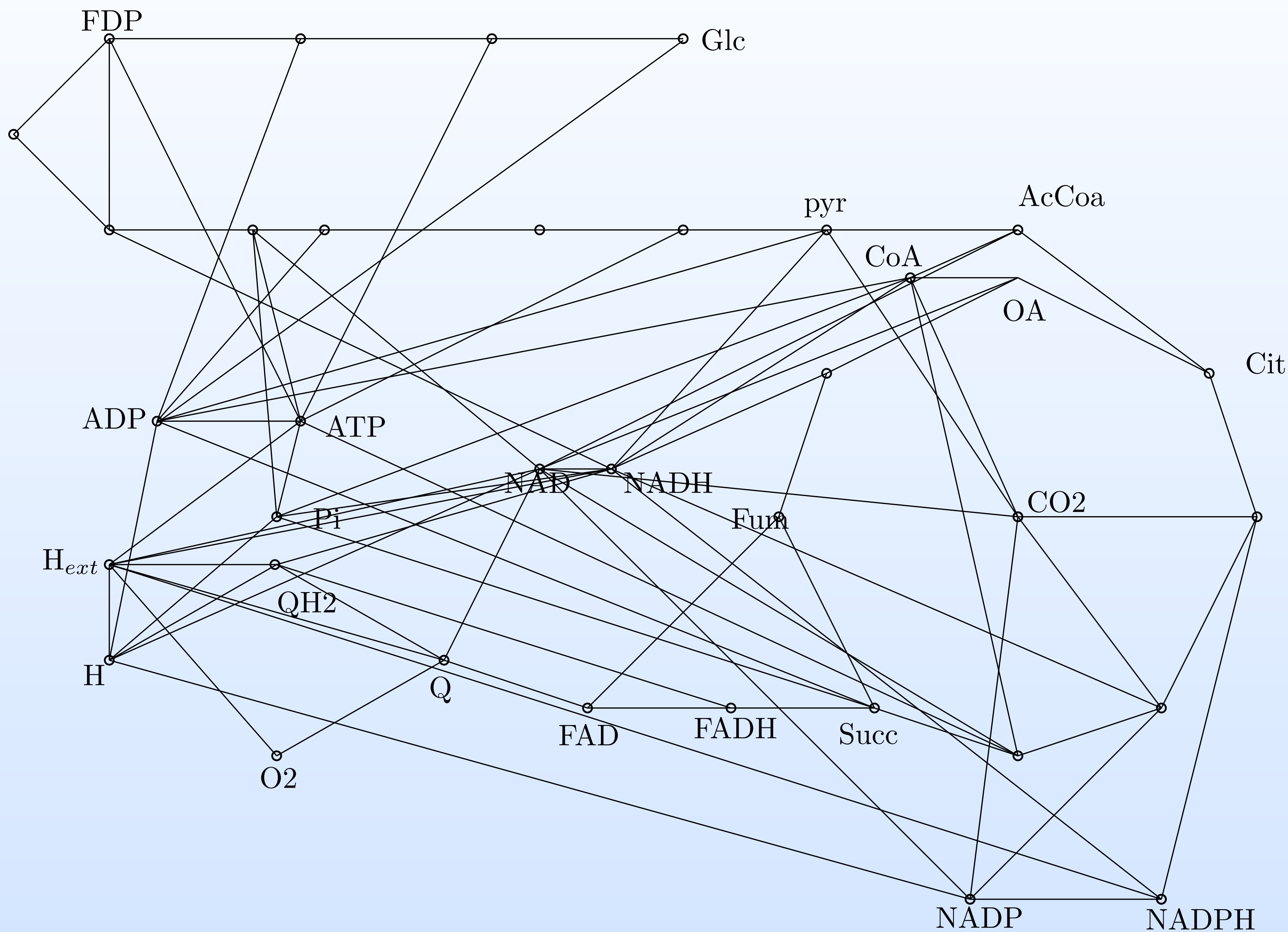
0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&+1&0&0&-1&+1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&+1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&+1&0&0&0&0&0&0&0&0&+1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&+1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-2&+2&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+2&-2&+1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+7&+5&0&-2&+4&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-7&-5&0&+2&-4&0&0
\end{bmatrix}
$$

Partial EColi metabolism

$$\overleftrightarrow{\nu} = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&+1&0&0&+1&+1&0&0&0&0&0&0&0&0&0&-6&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&+6&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&0\\
-1&0&-1&0&0&0&+1&0&0&+1&0&0&0&0&0&+1&0&0&0&0&0&0&0&-1&0&-1\\
+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
+1&0&+1&0&0&0&-1&0&0&-1&0&0&0&0&0&-1&0&0&0&0&0&0&0&+1&0&+1\\
0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&+1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&+1&+1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&+1&0&+1\\
0&0&0&0&0&-1&0&0&0&0&-1&0&0&0&-1&0&0&0&-1&+2&0&0&+1&0&0&0\\
0&0&0&0&0&+1&0&0&0&0&+1&0&0&0&+1&0&0&0&+1&-2&0&0&-1&0&0&0\\
0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&+1&0&0&-1&+1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&+1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&+1&0&0&0&0&0&0&0&0&+1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&+1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+1&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-2&+2&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+2&-2&+1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&+7&+5&0&-2&+4&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-7&-5&0&+2&-4&0&0
\end{bmatrix}$$

$$\begin{bmatrix} +1 & +1 & +1 & +1 & -1 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +5 & +6 & +2 & -2 & -17.25 & +1 & +21.25 \end{bmatrix}$$

Partial EColi metabolism



Decomposition in building blocks

$$
\begin{aligned}
S_1 &= NADH \\
S_2 &= CoA \\
S_3 &= P_i \\
S_4 &= NADPH \\
S_5 &= FADH \\
S_6 &= ADP \\
S_7 &= O \\
S_8 &= QH_2 \\
S_9 &= Succ \\
S_{10} &= C \\
S_{11} &= H
\end{aligned}
$$

Decomposition in building blocks

Example Decomposition:

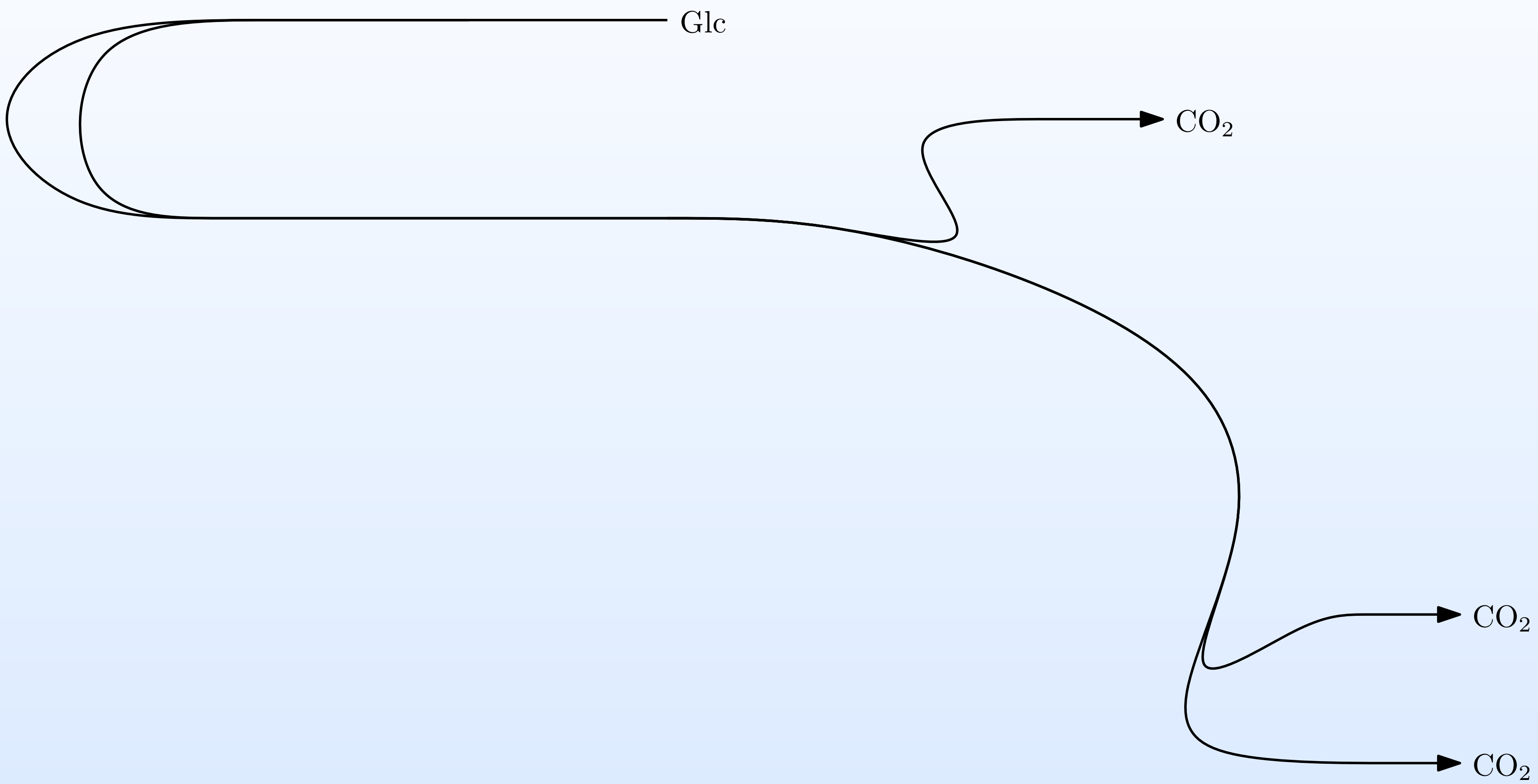
$$\begin{aligned}
Glc &= 6 \times (C) \\
O_2 &= 2 \times (O) \\
CO_2 &= 1 \times (C) + 2 \times (O) \\
ATP &= 1 \times (ADP) + 1 \times (P_i) \\
Cit &= 1 \times (Succ) + 2 \times (C) + 2 \times (O) \\
\ldots &
\end{aligned}$$

Decomposition in building blocks

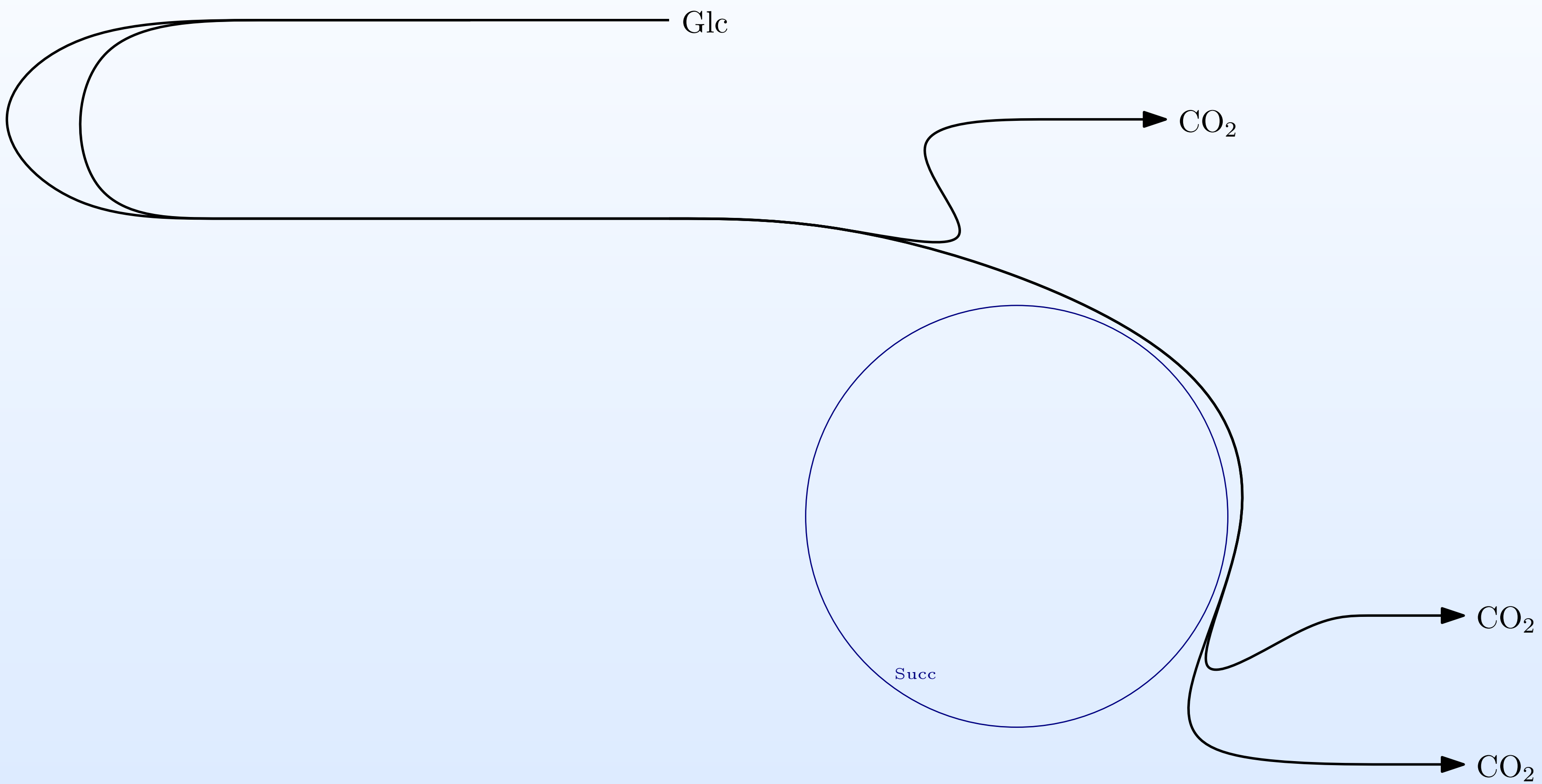
Submatrix for S_{10}:

	$R_1^{(10)}$	$R_2^{(10)}$	$R_3^{(10)}$	$R_4^{(10)}$	$-R_5^{(10)}$	$2R_6^{(10)}$	$2R_7^{(10)}$	$2R_8^{(10)}$	$2R_9^{(10)}$	$2R_{11}^{(10)}$	$2R_{12}^{(10)}$	$2R_{15}^{(10)}$	$2R_{16}^{(10)}$	$2R_{17}^{(10)}$	$2R_{18}^{(10)}$	$F^{(10)}$
$X_1^{(10)}$	0	0	0	0	0	0	0	0	0	0	$+2$	0	0	$+2$	$+2$	-6
$X_3^{(10)}$	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$+6$
$X_5^{(10)}$	$+6$	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$X_7^{(10)}$	0	$+6$	-6	0	0	0	0	0	0	0	0	0	0	0	0	0
$X_8^{(10)}$	0	0	$+6$	-6	0	0	0	0	0	0	0	0	0	0	0	0
$X_9^{(10)}$	0	0	0	$+3$	$+3$	-6	0	0	0	0	0	0	0	0	0	0
$X_{10}^{(10)}$	0	0	0	$+3$	-3	0	0	0	0	0	0	0	0	0	0	0
$X_{14}^{(10)}$	0	0	0	0	0	$+6$	-6	0	0	0	0	0	0	0	0	0
$X_{15}^{(10)}$	0	0	0	0	0	0	$+6$	-6	0	0	0	0	0	0	0	0
$X_{16}^{(10)}$	0	0	0	0	0	0	0	$+6$	-6	0	0	0	0	0	0	0
$X_{17}^{(10)}$	0	0	0	0	0	0	0	0	$+6$	-6	0	0	0	0	0	0
$X_{18}^{(10)}$	0	0	0	0	0	0	0	0	0	$+6$	-6	0	0	0	0	0
$X_{21}^{(10)}$	0	0	0	0	0	0	0	0	0	0	$+4$	-4	0	0	0	0
$X_{23}^{(10)}$	0	0	0	0	0	0	0	0	0	0	0	$+4$	-4	0	0	0
$X_{24}^{(10)}$	0	0	0	0	0	0	0	0	0	0	0	0	$+4$	-4	0	0
$X_{27}^{(10)}$	0	0	0	0	0	0	0	0	0	0	0	0	0	$+2$	-2	0

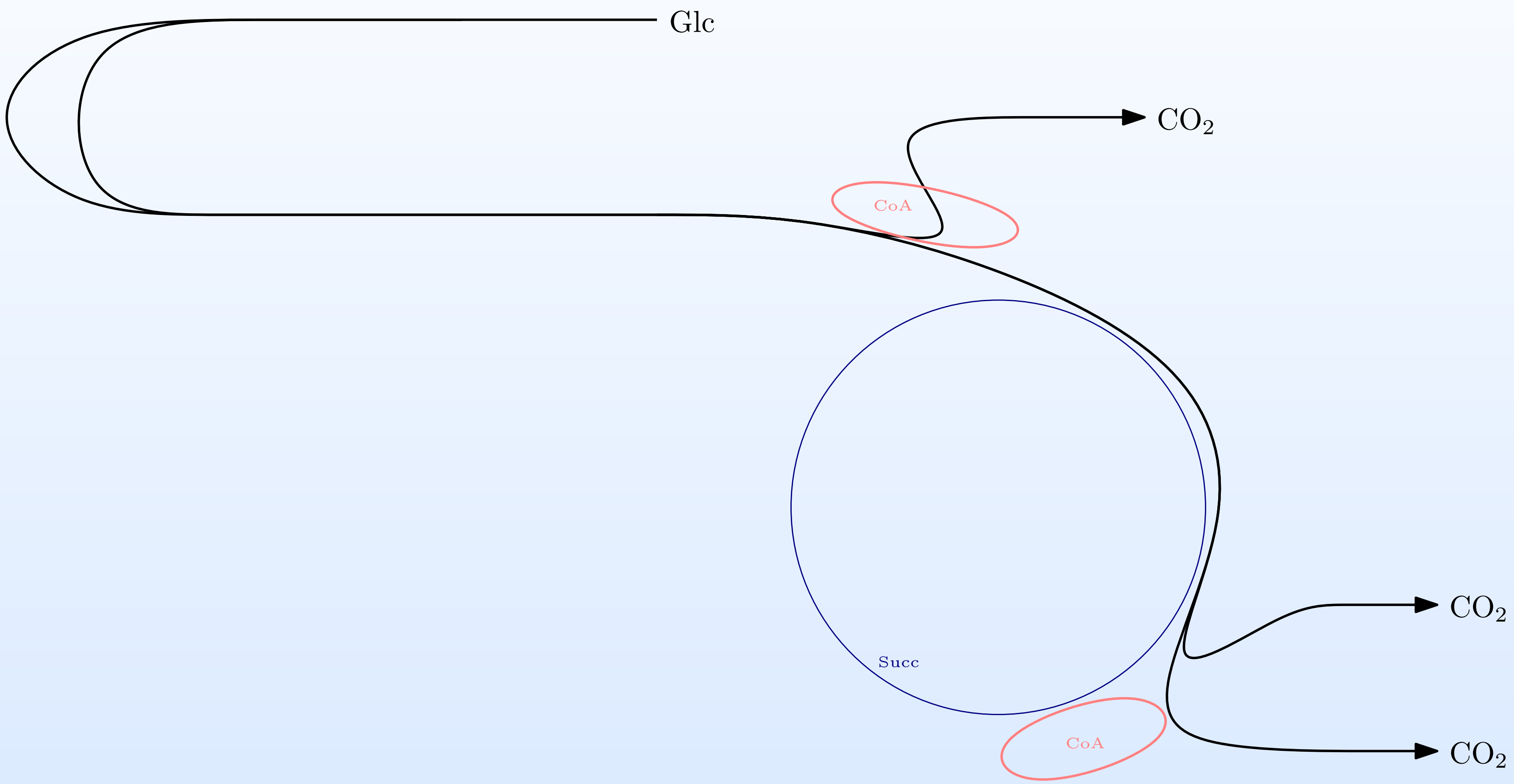
Network reconstruction



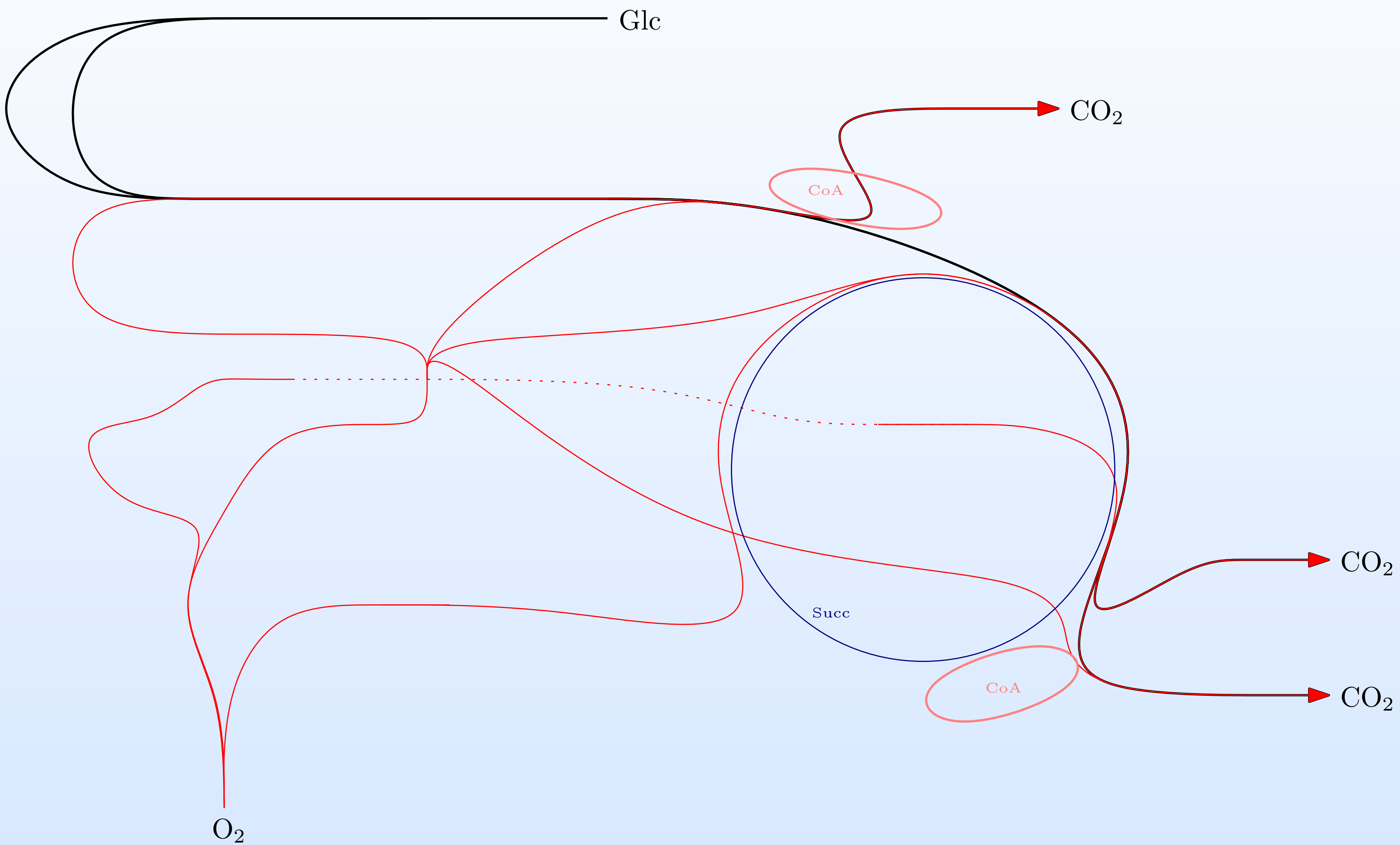
Network reconstruction



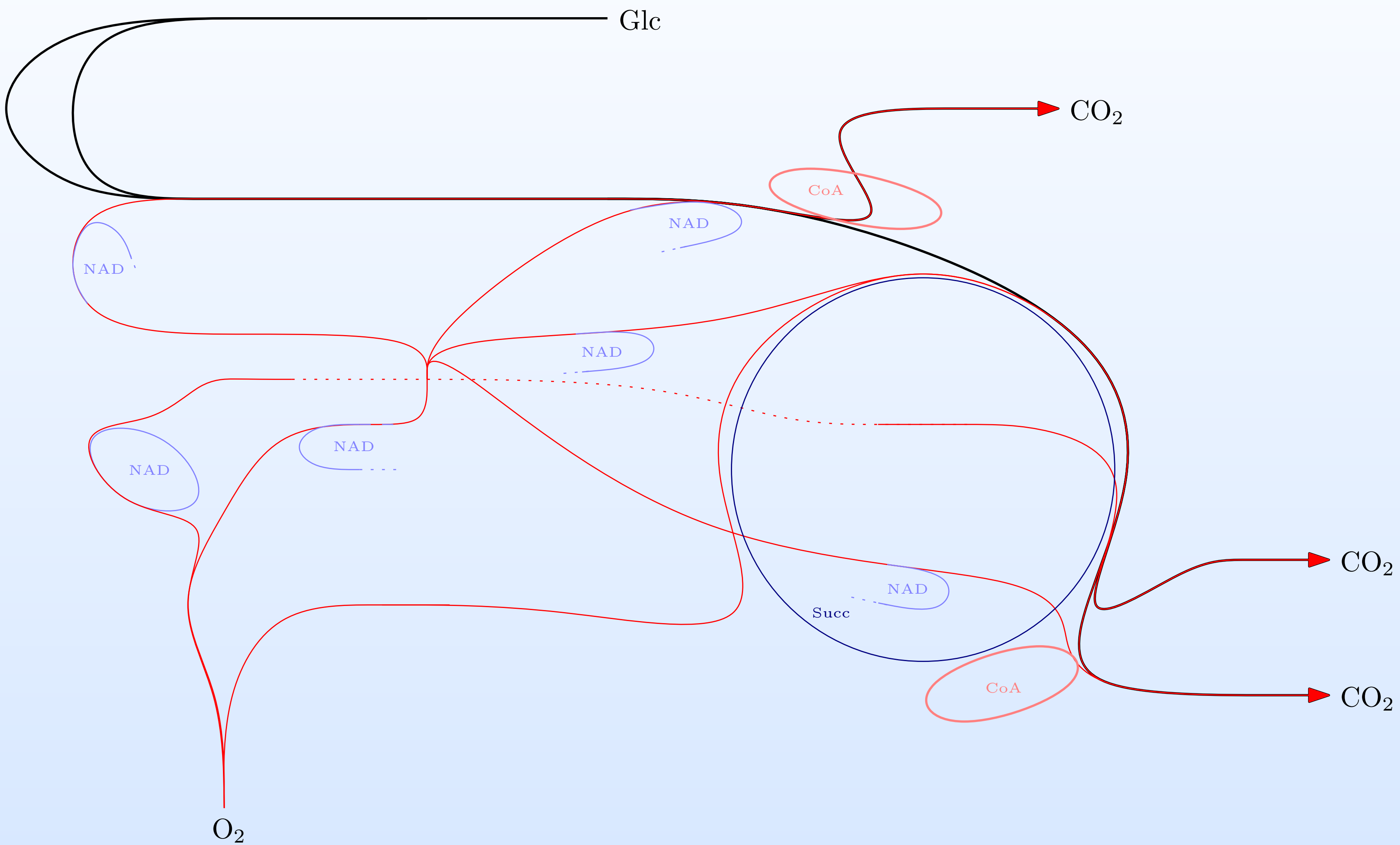
Network reconstruction



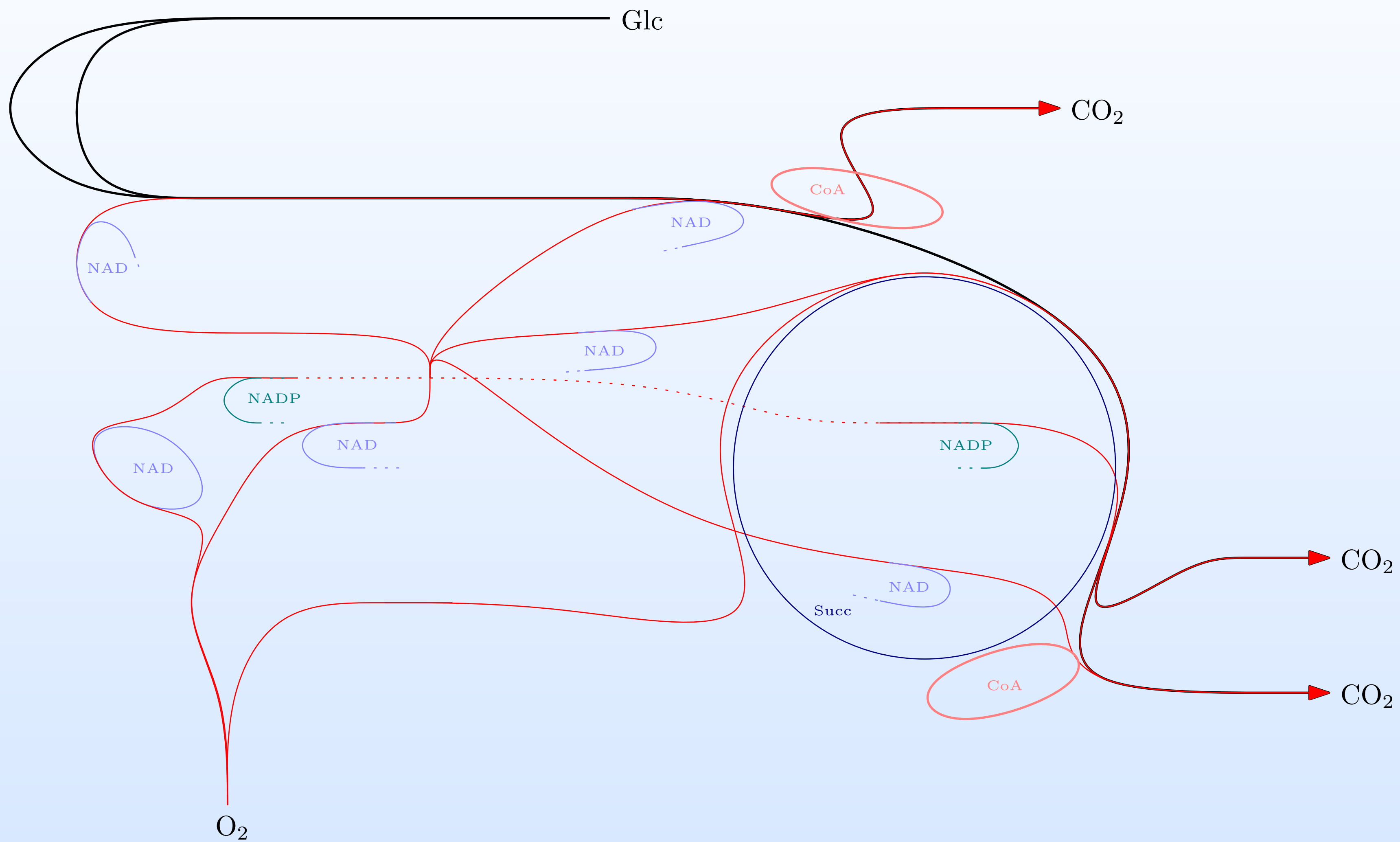
Network reconstruction



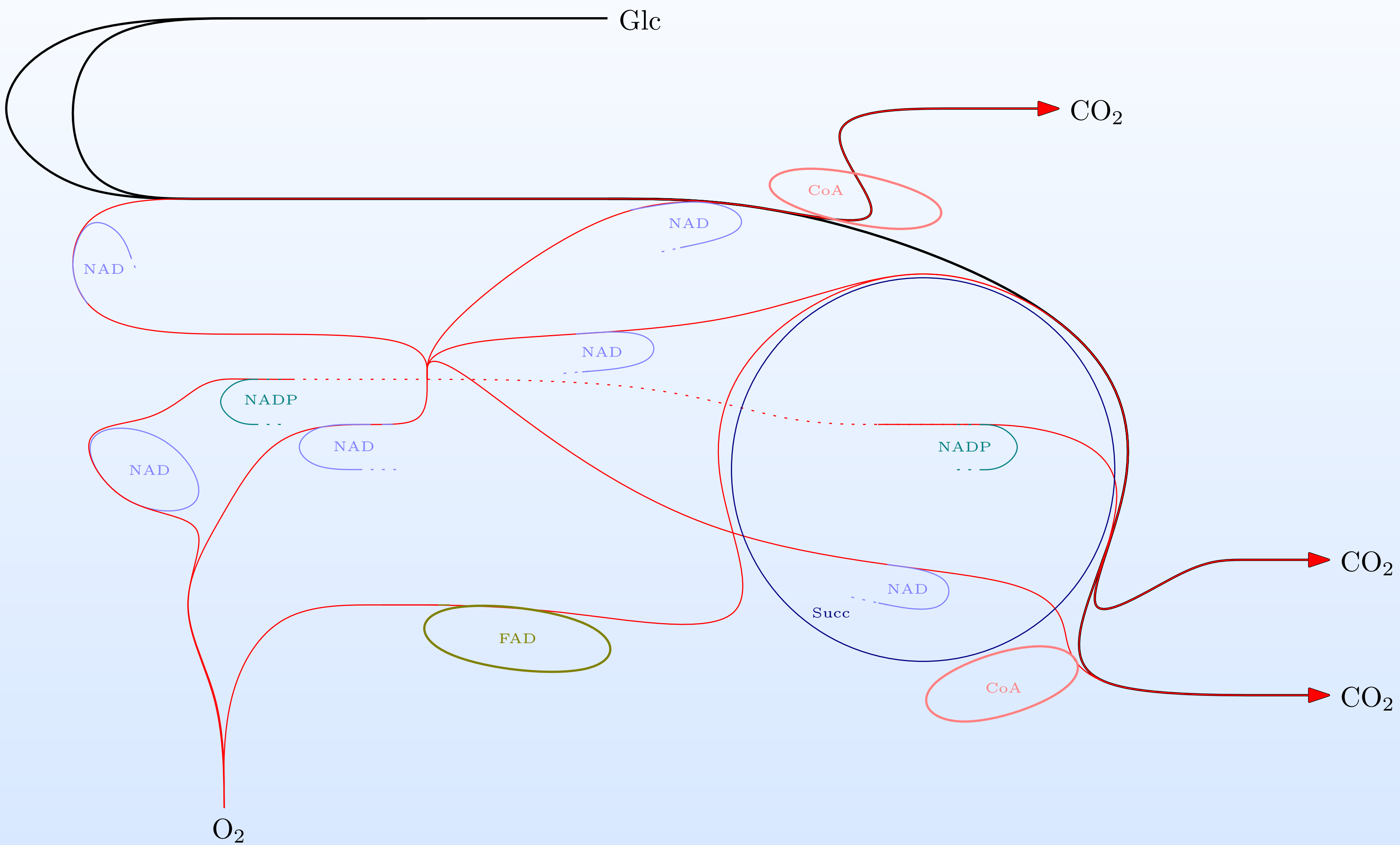
Network reconstruction



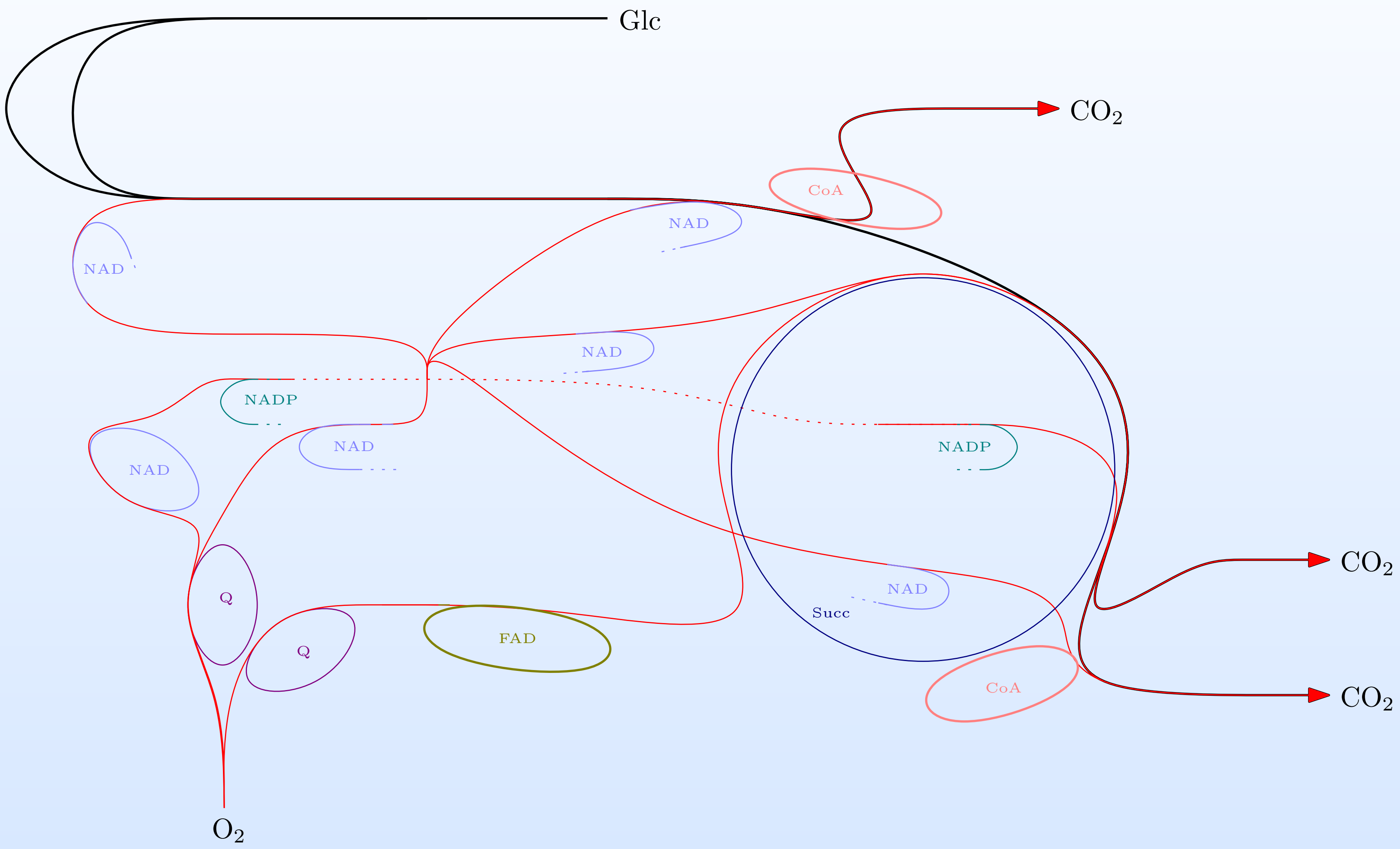
Network reconstruction



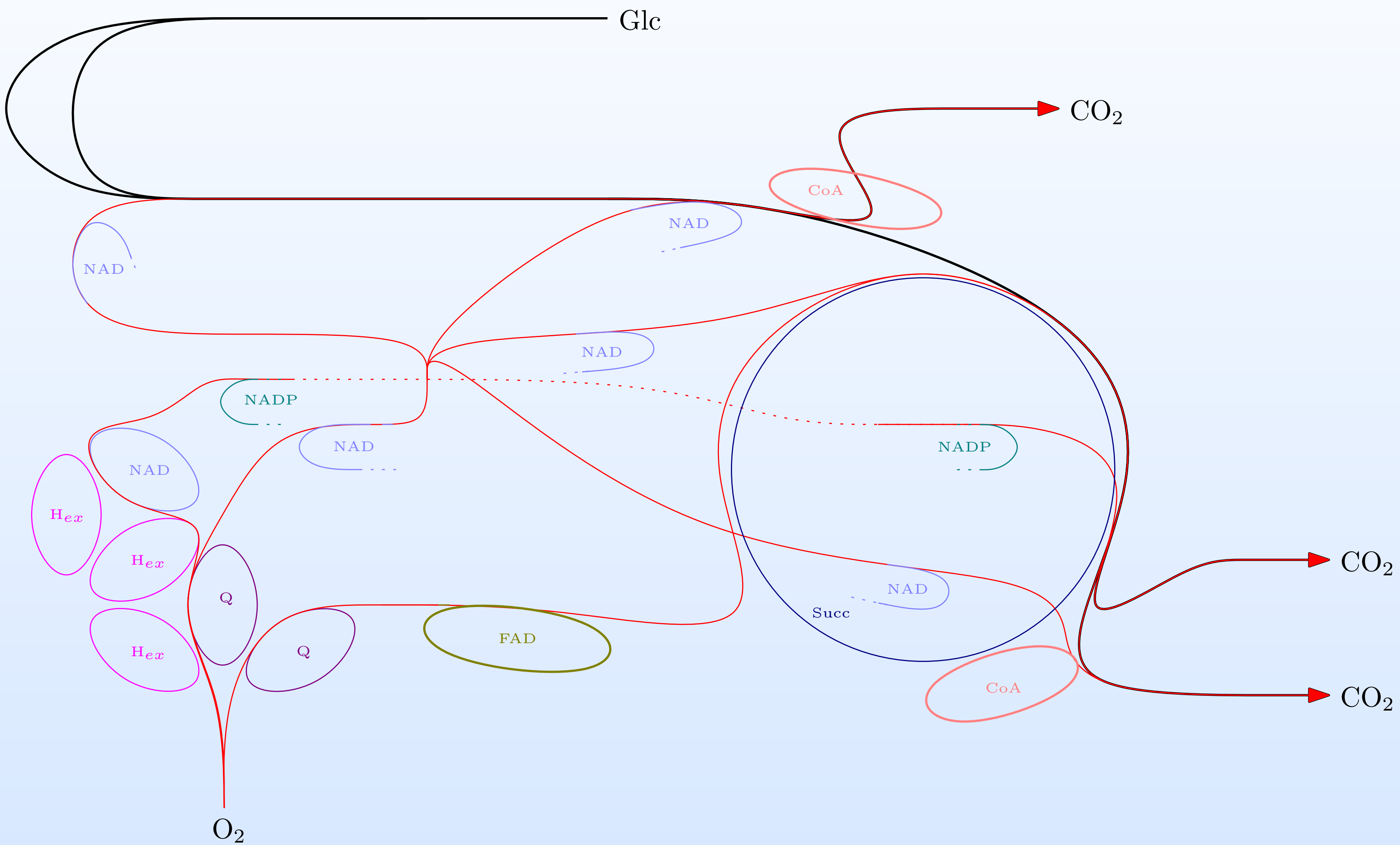
Network reconstruction



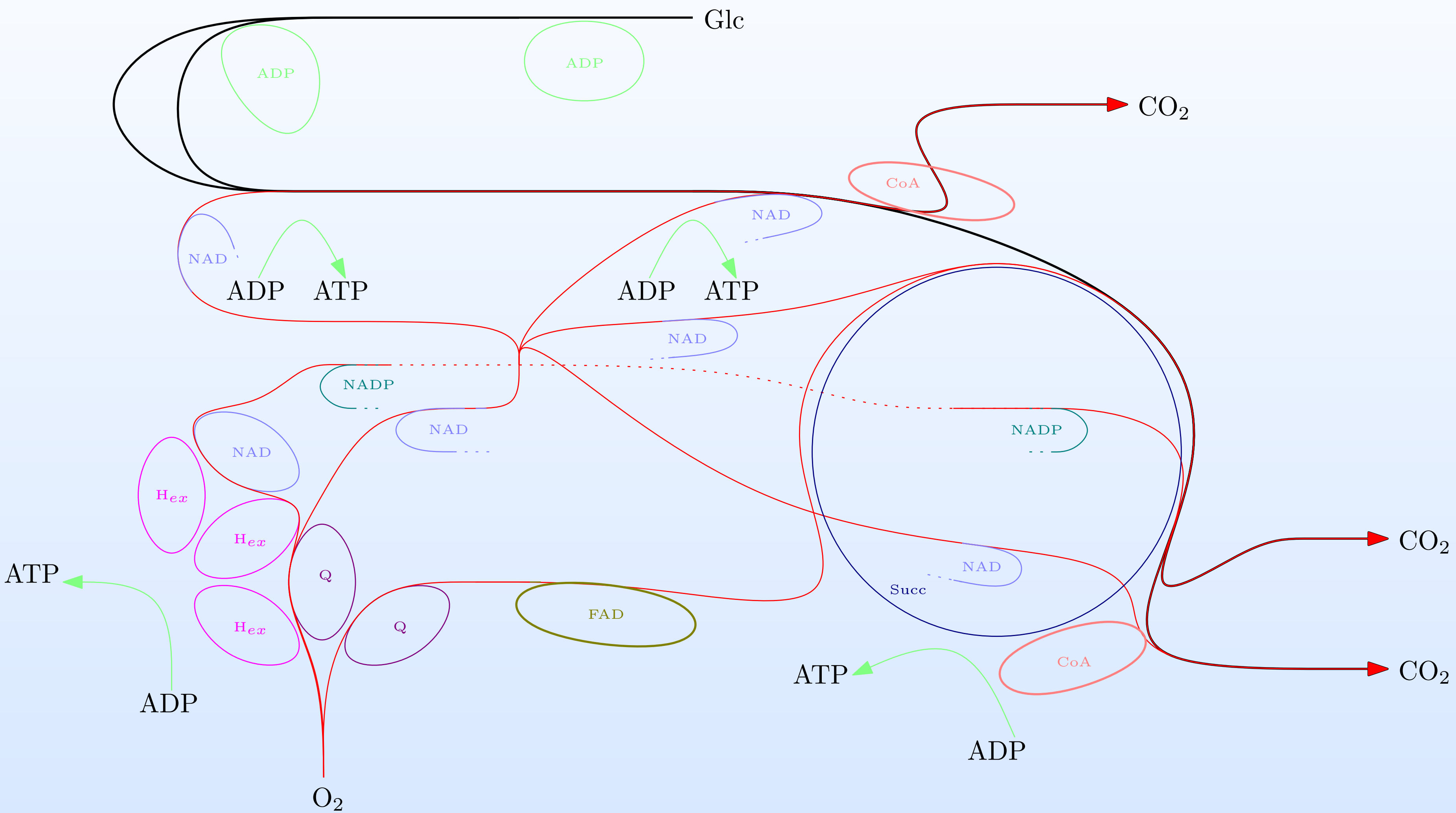
Network reconstruction



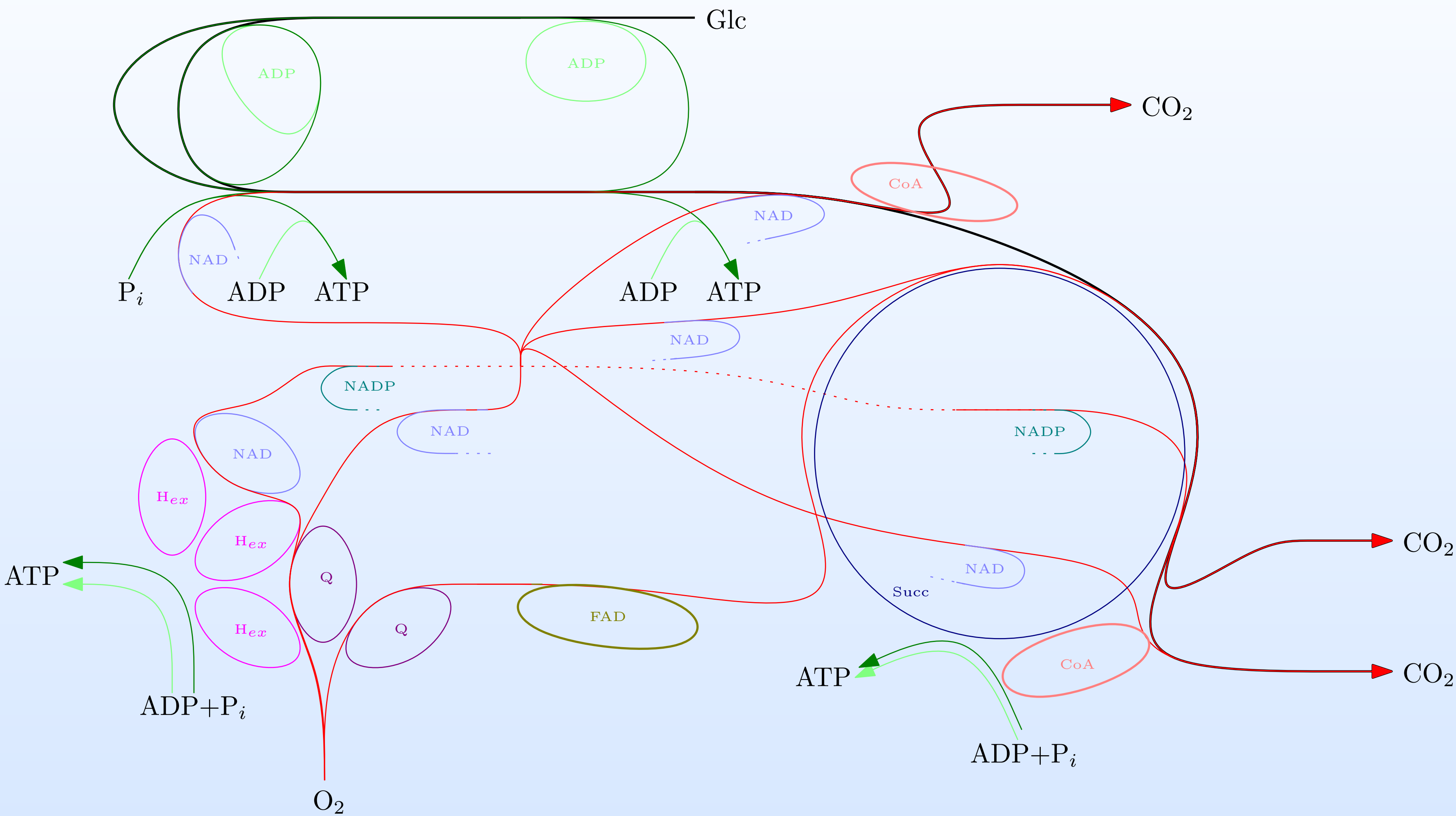
Network reconstruction



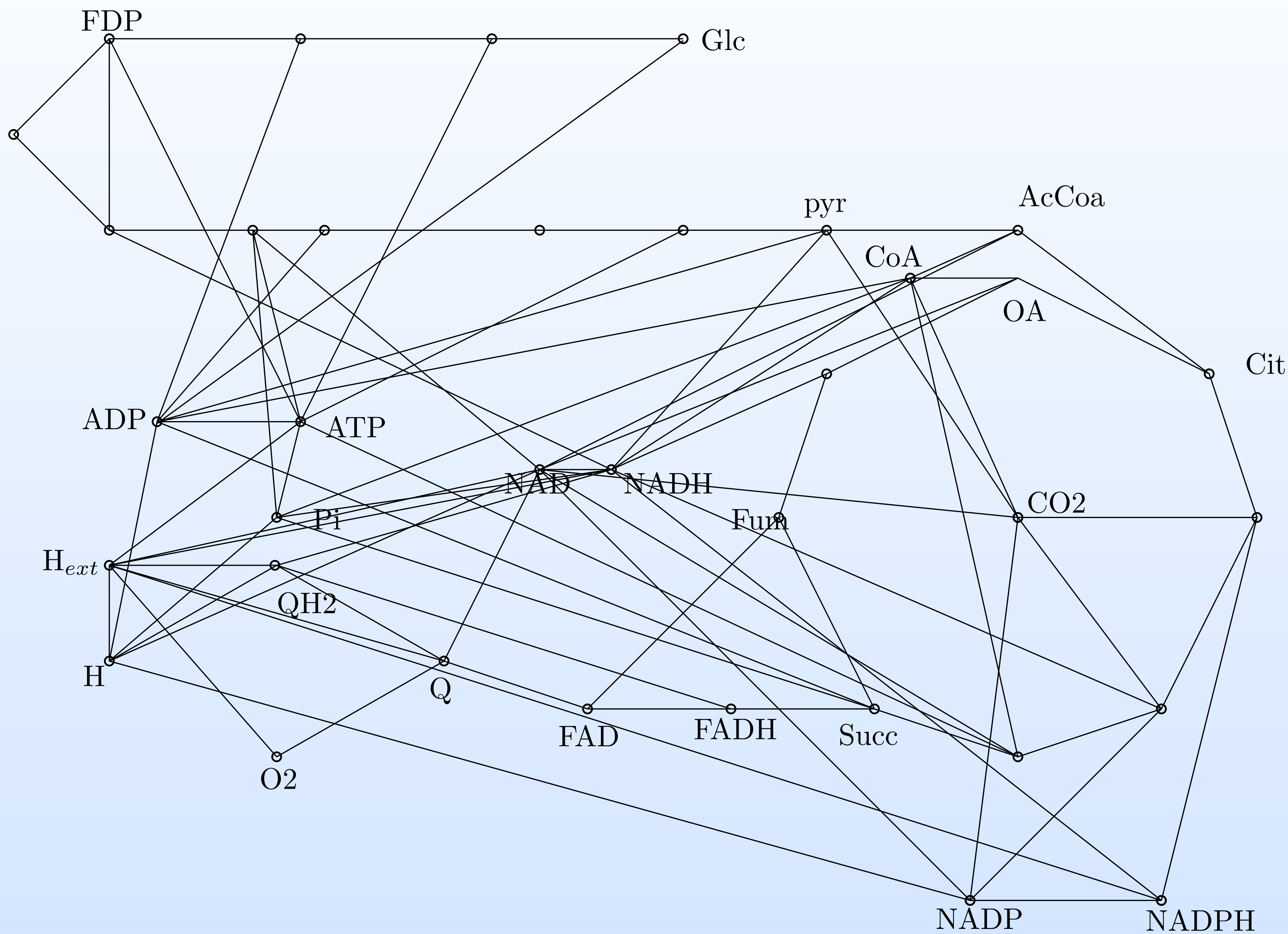
Network reconstruction



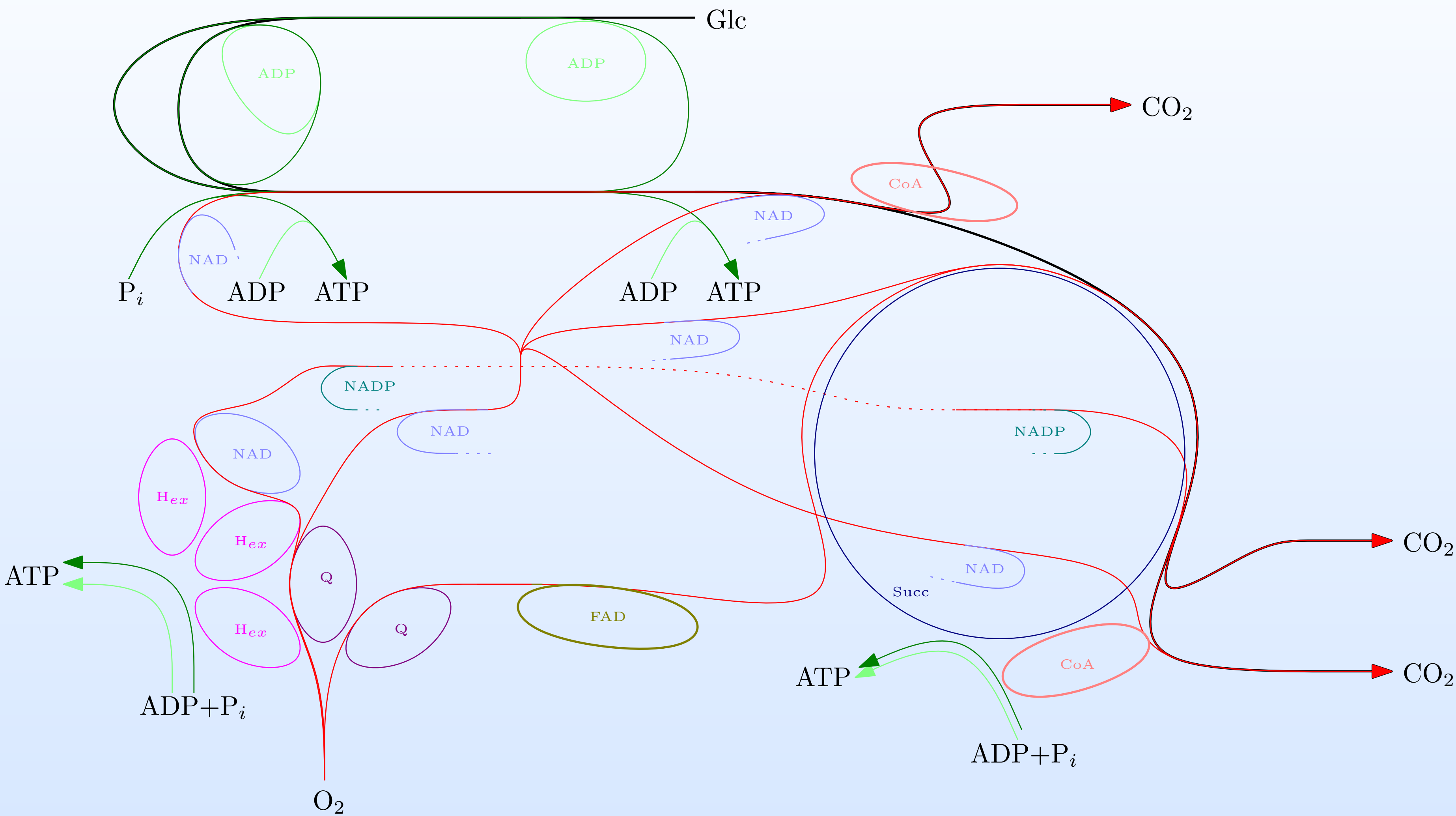
Network reconstruction



Network reconstruction



Network reconstruction



Conclusion

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- The global reaction network can be decomposed in a sum of subnetworks, each one relative to one building block.
- These subnetwork are smaller, and only composed of first order transformation. They are one-type node network.
- Any chemical reaction network can be described as a sum of linear fluxes and single loops, as the only elements able to be maintained in steady state.
- Self-organization in reaction network: linear fluxes as the onset of energy exchanges, and single loops as emergent (auto)catalysis.