# Organization of Complex Networks

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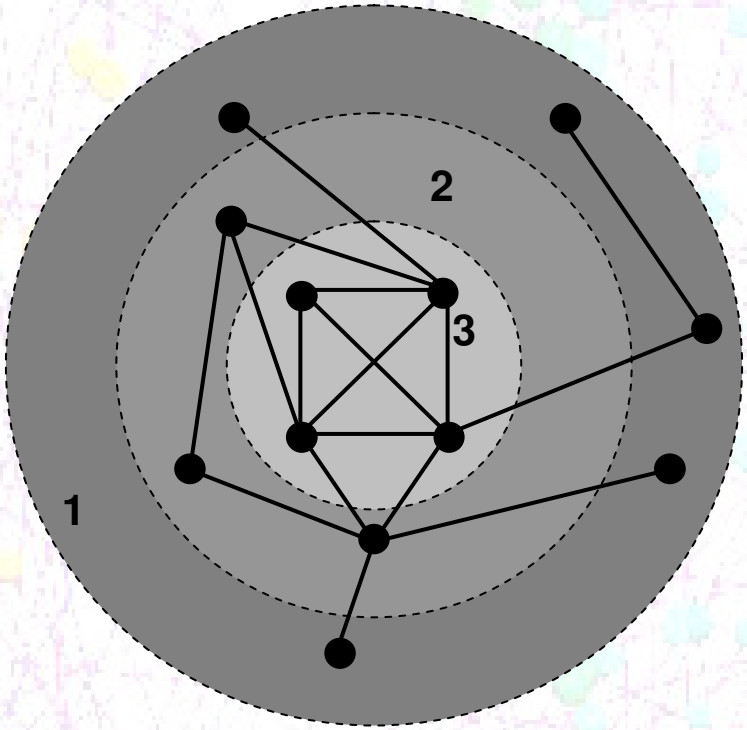
# K-cores: Definitions, Extraction Rules

**K-core is the sub-graph with nodes of degree at least k within the sub-graph.**

**Pruning Rule:**

1) Remove from the graph all nodes with k=1.
2) Some remaining nodes may now have new k = 1 nodes.
3) Repeat until there is no nodes with k = 1.
4) The remaining network forms the 2-core.
5) Repeat the process for higher k to extract other cores




**K-shell is a set of nodes that belongs to the K-core and NOT to the K+1 core**

**K-crust is a union of all shells with indices <= k**

**Any network can be represented as a number of self-enclosed K-cores!**

# Topology Analysis: Some Facts, Questions and Answers

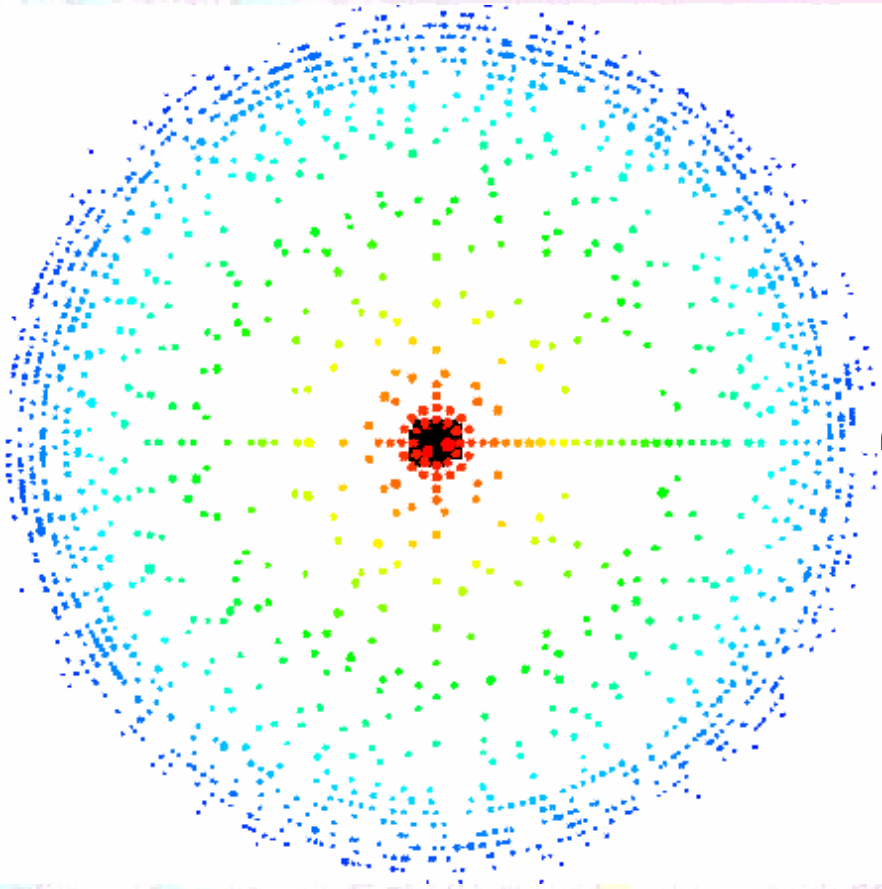
- Understanding the topology of a Complex System is often an important step towards understanding its function.
- Many Systems are extremely large: $N \sim 10^4 - 10^7$
- Robust techniques are required for exploration of large systems.
- What characteristics/techniques are employed in studies of network topology?

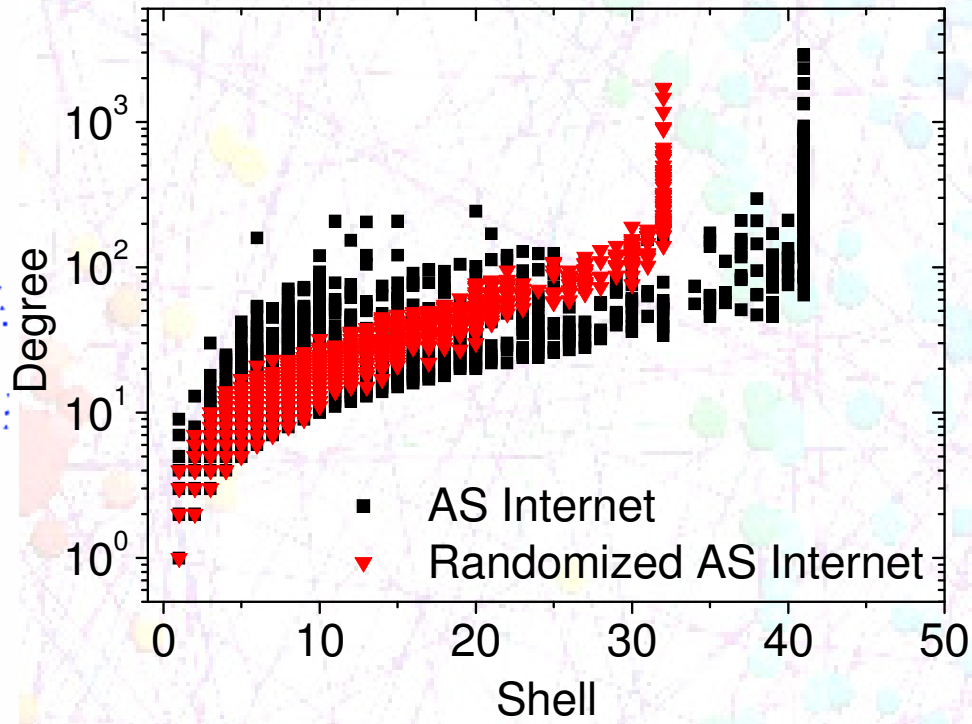
*Node degree, Clustering, Cliques, Centrality, Modularity, K-cores…*

K-core is the sub-graph with nodes of degree at least k within the sub-graph.

- What are the features of the k-core analysis?
- K-core analysis promotes node degree to a global property of a network
- K-core analysis allows *natural* representation of a network as a set of layers.
- K-core calculation is fast $O(N)$ compared to other global characteristics.

# The Internet: Autonomous System Level : K-core structure

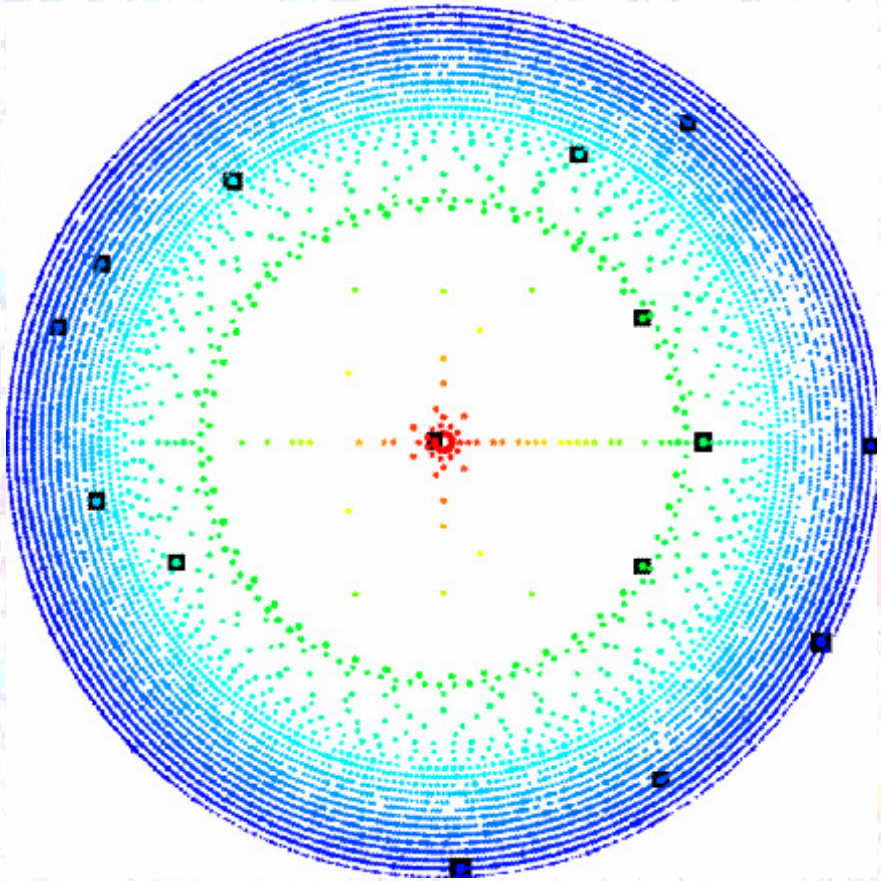


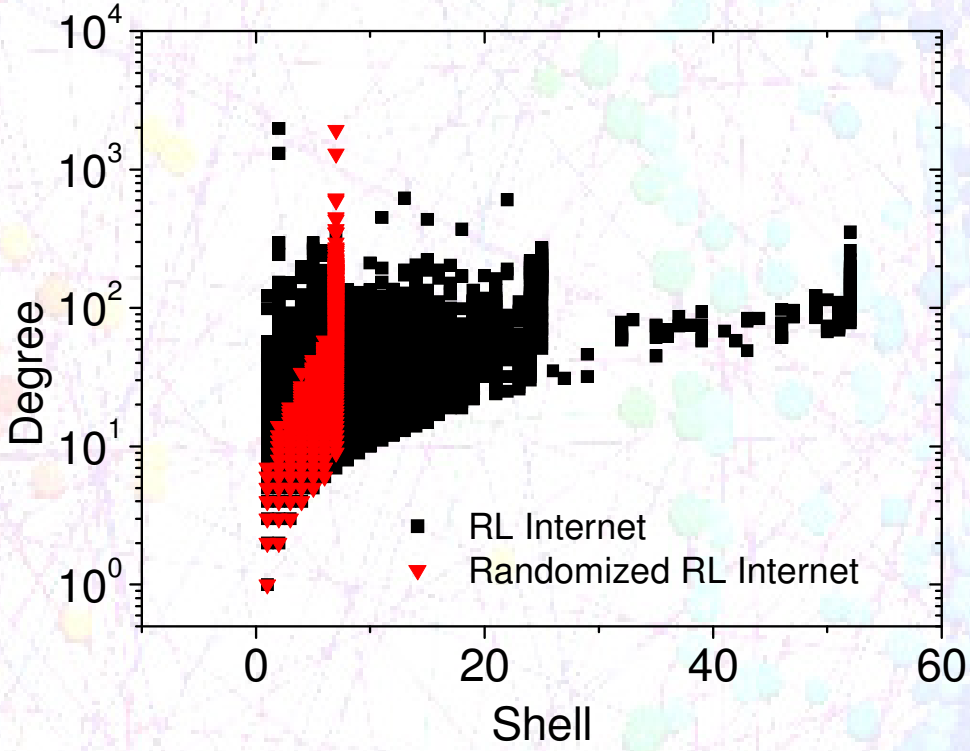



**Randomized version of the AS Internet has smaller number of k-shells.**

**Nodes with larger degrees are typically located in higher k-shells.**

# Internet at the Router Level: K-core structure

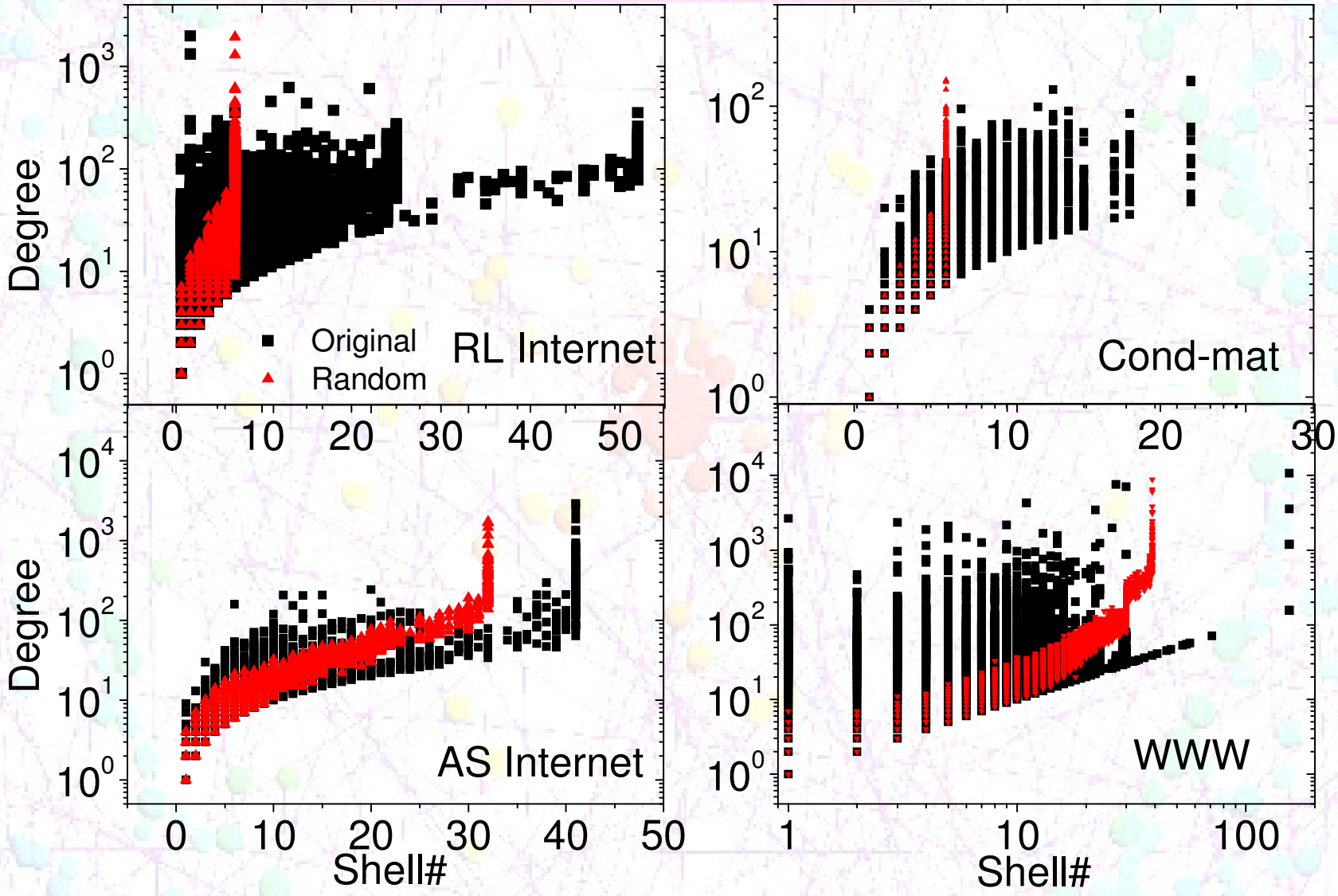





**Randomized version of the RL Internet has very few shells!**

**Nodes with larger degrees are typically located in small k-shells!**

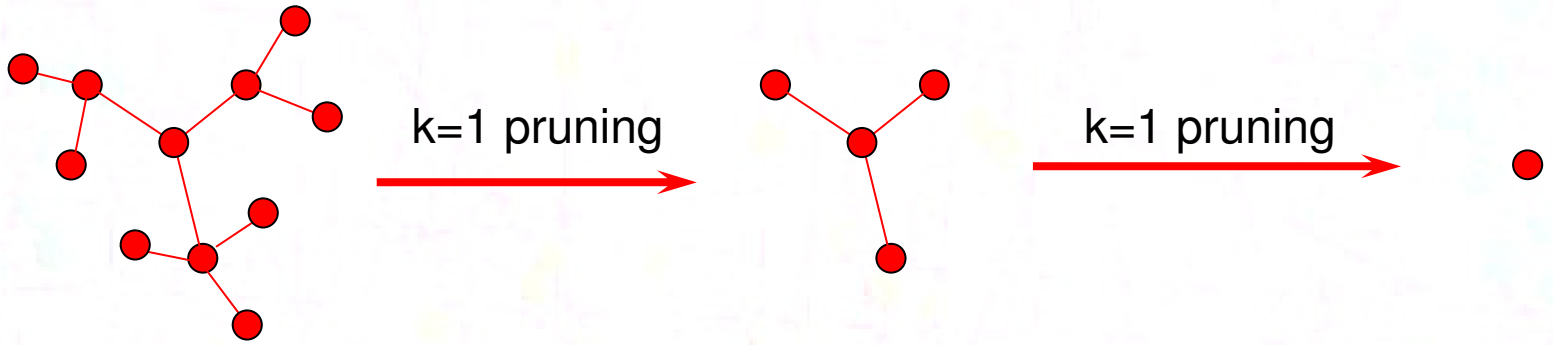
# Are K-shells Fully Determined by Degree Distribution?



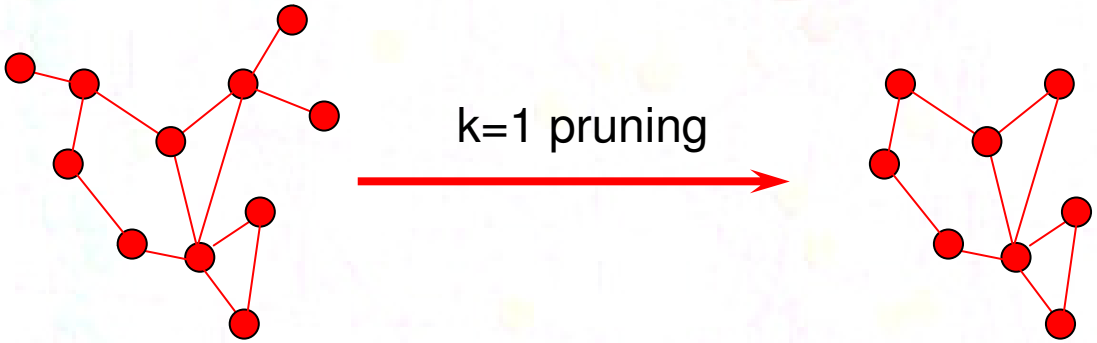

**No! K-shells depend on the way nodes are connected in the network.**

# Hypotheses

**1) K-shell structure depends on clustering:**




Tree-like networks are entirely decomposed at the k=1 step.



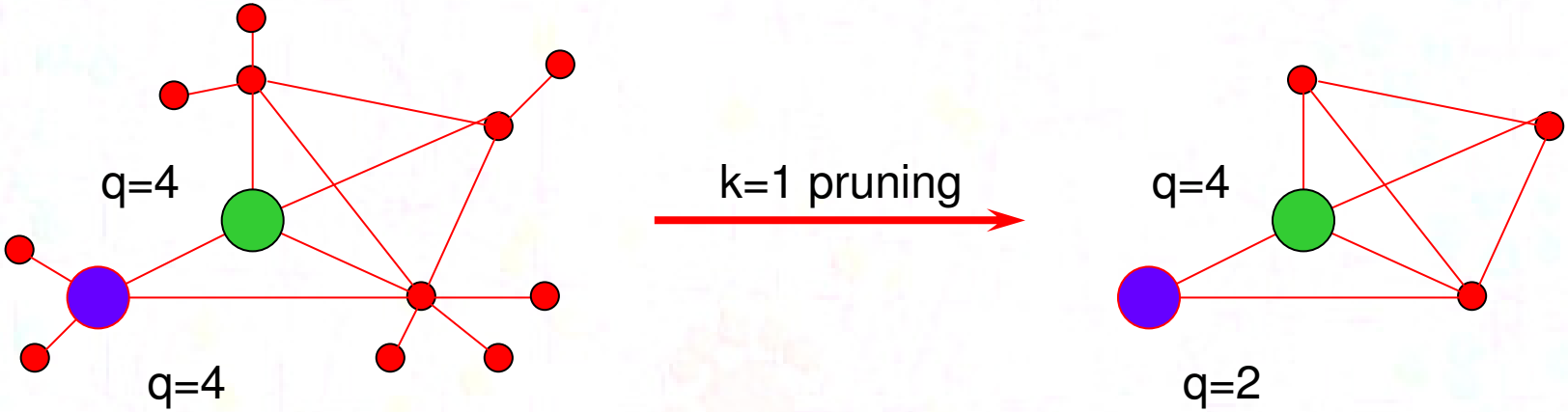

Loops are “vital” for the k-shell structure.

Triangle is the simpliest loop. We use clustering as a measure of density of loops.

**Networks with higher clustering should have more shells**

# Hypotheses

**2) K-shell strucrure depends on degree-degree correlation:**



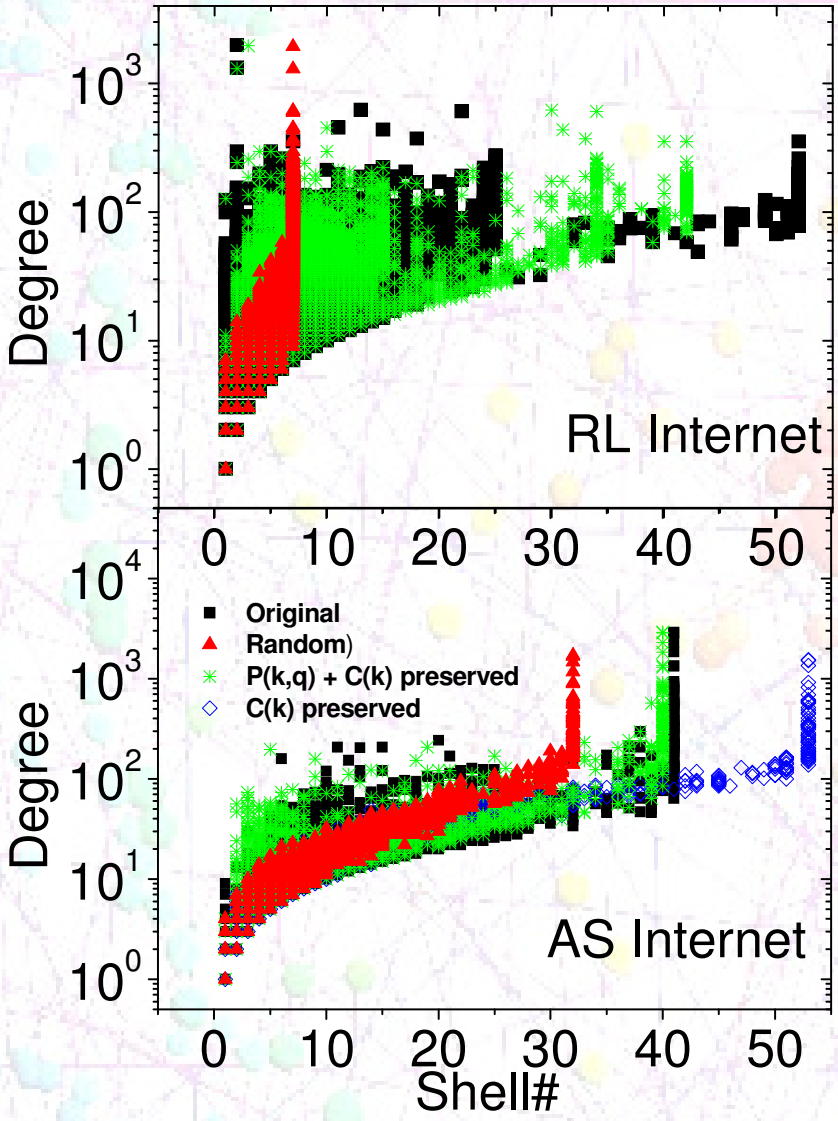

Hubs, mostly connected to small degree nodes tend to loose their links faster under k-pruning. (Compared to hubs that connect to other hubs.)

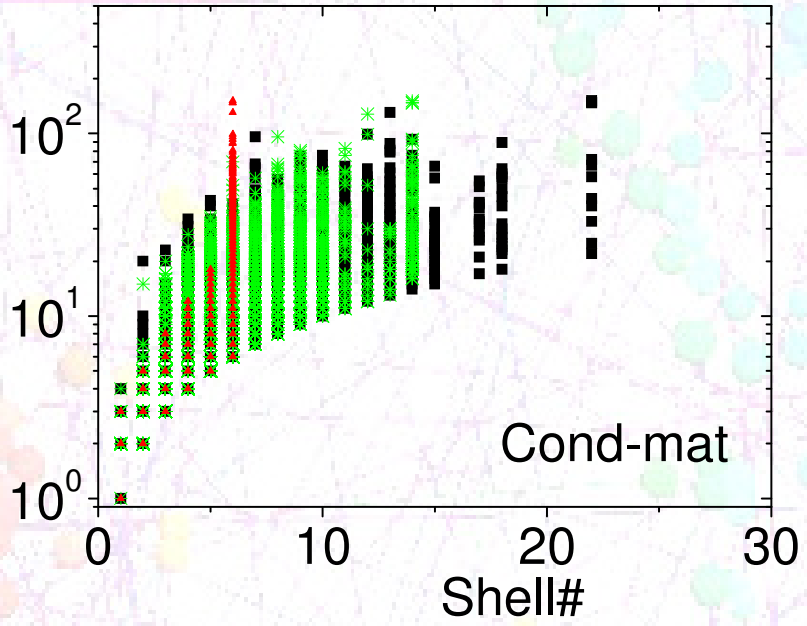
**Assortative Networks are resilient under k-pruning and should have more shells!**

What happens to the k-shell structure if one preserves both

$P(q_1, q_2)$ and $C(q)$ ?

# What if we preserve Degree Correlation and Clustering?




Both Assortativity and Clustering seem to increase the total number of k-shells.

Preserving only clustering allows to achieve Greater number of k-shells in AS internet due to its disassortativity.

*M.Kitsak, L.K. Gallos, S. Havlin, H.A. Makse, and H.E. Stanley, "Organization of Complex Networks " (in preparation) (2008).*

# Summary and Conclusions

- **K-core analysis is a powerful tool for the analysis of global network structure.**

  *K-shell structure promotes the degree concept to the global level Network is covered with a set of naturally emerging k-shell layers.*

- **Real Networks have non-trivial k-shell structure which significantly deviates from random models.**

  *The analysis of the k-shells of real networks allows one to reveal new aspects of the network topology.*

- **The k-cores appear to be strongly affected by degree correlation and clustering in networks.**

  *Assortativity and clustering level strongly affect the k-cores (k-shells) in the network.*

# Credits

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