

# Non-equilibrium Process of the Random Exchange Model

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*Talk at NetSci'08*

2008—06—27

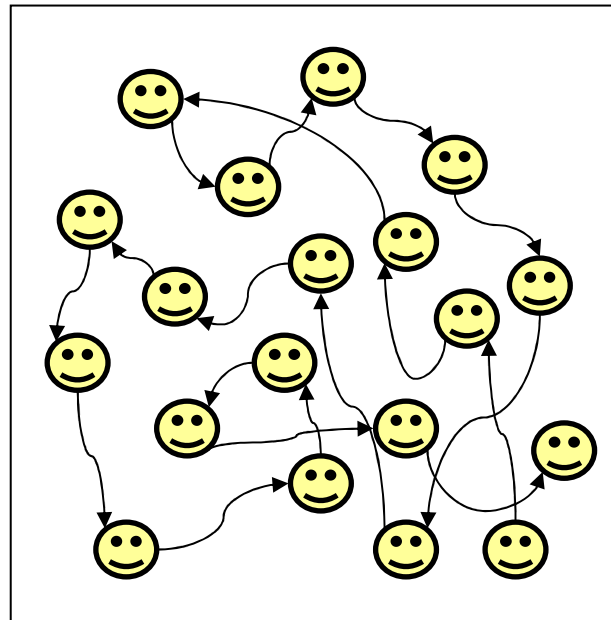
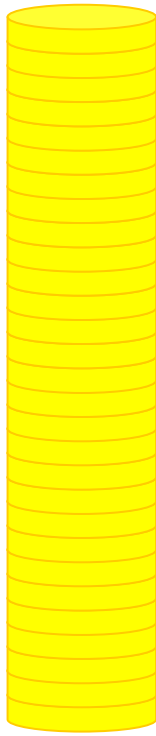
# Purpose

- With the help of random exchange process, we demonstrate the non-equilibrium process of money creation, i.e., the realizing process of money multiplier.
- The characteristics of this kind of process.
- The source of money creation.

# Random exchange models

- *S. Ispolatov, P. L. Krapivsky and S. Redner, Eur. Phys. J. B 2, 267(1998).*
- *A. Drăgulescu and V. M. Yakovenko, Eur. Phys.J. B, 17, 723 (2000).*
- *A. Chakraborti and B. K. Chakrabarti, Eur.Phys. J. B, 17, 167 (2000).*
- .....
- *B. Hayes, Am. Scientist 90, 400(2002).*

# Random exchange models



Money is always transferred from one to another in the trading process.

# Applications of these models

- Monetary Distribution
- Money Circulation
- Economic Mobility
- Money Creation

# Random exchange models with debt

- ☆ R. Fischer, D. Braun, Physica A (2003)  
321, 605--618.
- ☆ N. Xi, N. Ding, Y. Wang, Physica A (2005)  
357, 543--555.
- Stationary monetary distribution——  
asymmetric Laplace distribution
- Weakness: the source of money,  
nonequilibrium process

# In economics

- Credit and endogenous money
- The process of money creation
- Multiplier of money

$$m = \frac{M}{M_0}$$

- Weakness: static description on the relation between money aggregate and relevant parameters.

# An Exchange Model with a Bank

- Structure
  - N agents and one virtual bank
- Initial conditions
  - $m_0$ ,  $M_0 = m_0 \times N$
- Trading rule
  - randomly matching N pairs for trading in each turn
  - $\Delta m = 1$  will be transferred between each pair of traders.
  - For agents, it is allowed to borrow money from the bank
- Constraints
  - The total volume of bank loan  $D$

# Random Walk

- Without any constraints, random exchange yields free transfer of money among agents, so that there is no barrier for the money individual agents hold to become more or less.
- In any one trade, if the number of agents who have amount of  $m_i$  decreases, there must be an increase of number of agents who hold amount of  $m_j$ .

Thus, the change in the number of agents in our model can be described as a conjunct random walk.

# Non-equilibrium—Normal Diffusion

- As the density function of monetary wealth distribution is given by  $p(m,t)$ , the diffusion equation can be expressed as

$$\frac{\partial p}{\partial t} - \tilde{D} \frac{\partial^2 p}{\partial m^2} = 0 \xrightarrow[\Delta m = 1]{\tilde{D} = 1} \frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial m^2} = 0$$

Initial condition

$$p(m,0) = \delta(m - m_0) = \begin{cases} 1 & (m = m_0) \\ 0 & (m \neq m_0) \end{cases}$$

The final solution

$$p(m,t) = \frac{1}{\sqrt{4\pi t}} e^{-\frac{(m-m_0)^2}{4t}}$$

# The increase of money in the non-equilibrium process

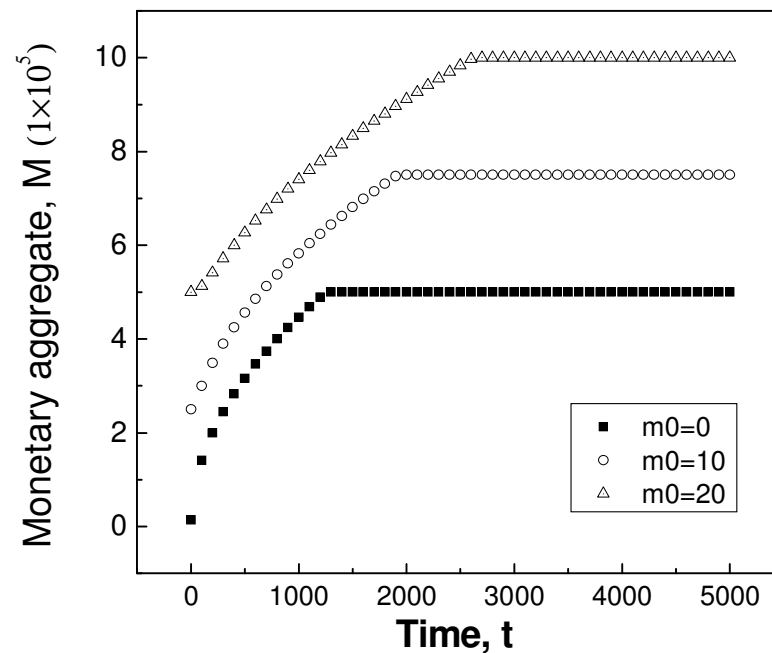
- Money aggregate at time  $t$

$$M(t) = \int_0^{\infty} Nmp(m, t)dm = \frac{N\sqrt{t}}{\sqrt{\pi}} e^{-\frac{m_0^2}{4t}} + \frac{m_0 N}{2} [1 - \operatorname{erf}(-\frac{m_0}{2\sqrt{t}})]$$

- Without limitation on bank loan, money aggregate will increase to infinity, i.e.,

$$M^* = \lim_{t \rightarrow \infty} M(t) = \infty$$

# Evolution of Money Aggregate



The limit of  $D=5 \times 10^5$  is set upon the total volume of bank loan

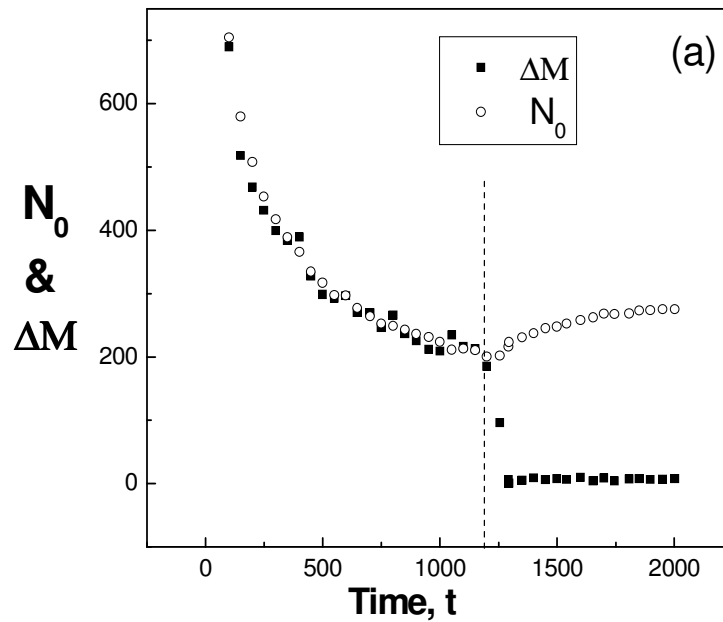
# The source of money creation

- Variation of money aggregate

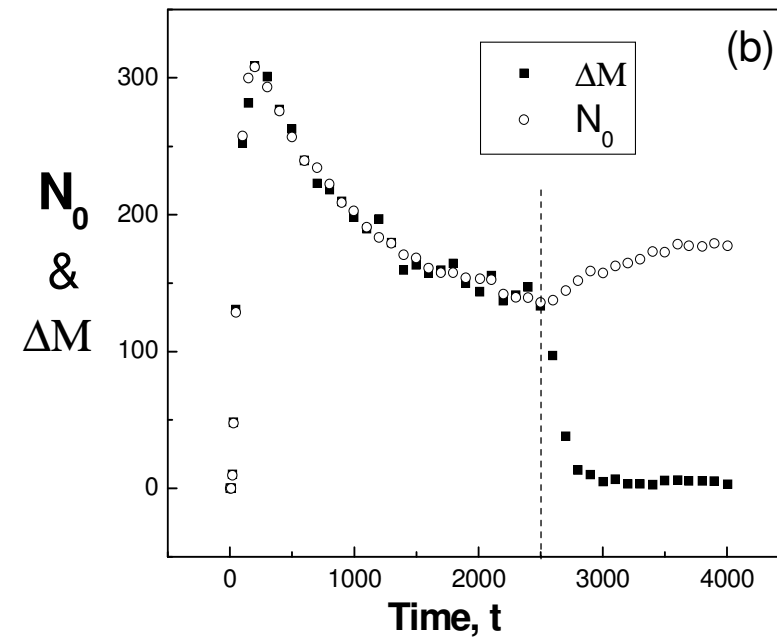
$$\frac{dM}{dt} = N \int_0^{\infty} m \frac{d}{dt} p(m, t) dm = N \times p(0, t) = N_0(t)$$

This result indicates that the increase in money aggregate is the number of agents who has no money and no debt.

# The correlation between increment of money and $N_0$



(a)  $m_0 = 0$  , (b)  $m_0 = 20$



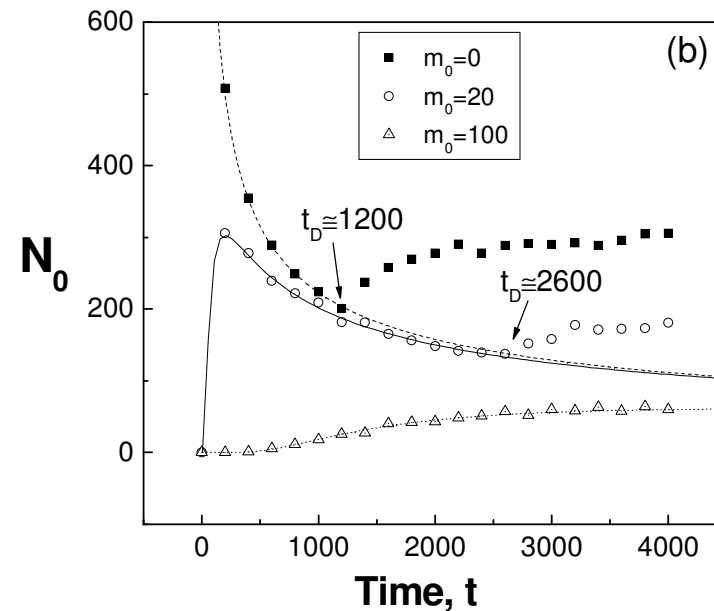
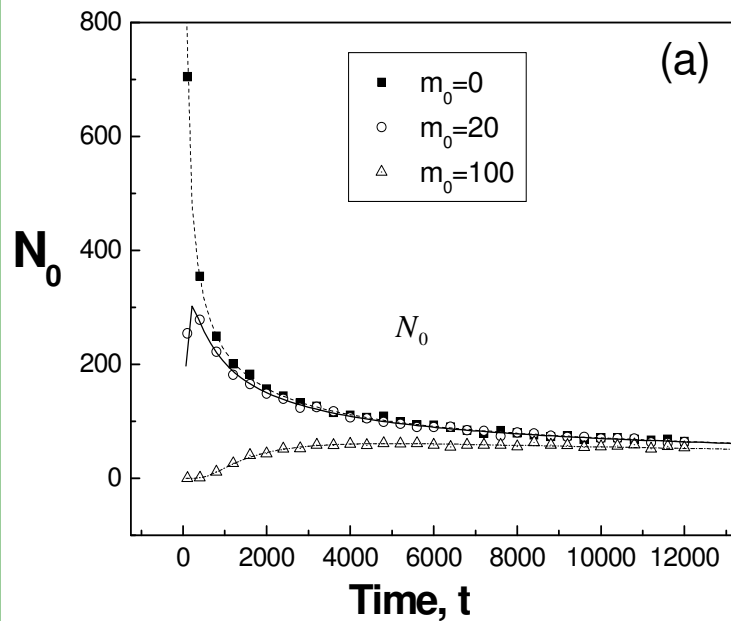
# The evolution function of $N_0$

- The number of agents whose amount of money is none

$$N_0(t) = N \times p(0, t) = \frac{N}{\sqrt{4\pi t}} e^{-\frac{m_0^2}{4t}}$$

From this result, we can see that the function of  $N_0$  in terms of time  $t$  is dependent on the initial amount of money. However, as time is long enough, all the evolutions will converge to one common relationship.

# The evolution of $N_0$



(a) Without limitation of bank loan; (b) The limit of bank loan  $D = 5 \times 10^5$

The dots are simulation results, the curve are theoretical predictions

# Conclusions

- Money creation is executed during a non-equilibrium process of random trading
- The random transfer of money has analogy to random walks and the process can be described by a diffusion equation.
- The increase in money aggregate arises from the number of agents who have none.



**Thanks!**



Your comments are welcome.