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Cascades on correlated and modular networks

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Overview

An analytical approach to determining the mean avalanche size in a broad class of dynamical models on random networks is introduced. Previous results on percolation transitions and epidemic sizes are shown to be special cases of the method. The time-dependence of cascades and extensions to networks with community structure or degree-degree correlations are discussed. Analytical results for the rate of spread of innovations in a modular network and for the size of k-cores in networks with degree-degree correlations are confirmed with numerical simulations.

Introduction

In certain dynamical models on complex networks, interactions between the nodes (vertices) of the network may cause initially localized effects to propagate throughout the whole network. Such *avalanches* or *cascades* occur, for example, in the transmission of infectious diseases through communities and of computer viruses over email networks. The adoption of innovations and the spread of fads may be modeled as cascade processes on social networks (Watts, 2002), with the aim of viral marketing being the creation of global cascades on these networks (Watts and Dodds, 2007).

The dynamics of cascades are strongly dependent upon the topological structure of the underlying network and on the details of how the cascade spreads among the nodes of the network. In the class of examples considered here, each node of the network can be in one of two states, either *active* (also termed *damaged* or *infected*) or *inactive* (*undamaged* or *susceptible*), with nodes updating their states depending on the number and state of the node's immediate neighbors in the (undirected) network. Networks are chosen from an ensemble of graphs with specified degree distribution (i.e. using the configuration model (Newman *et al*, 2001)), and both synchronous and asynchronous updating may be considered. We show that for a class of such models the average cascade size may be determined analytically (averages being taken over an ensemble of realizations) (Gleeson and Cahalane, 2007). This basic model is also extended to networks with strong community structure or with degree-degree correlations. Previous results on percolation and k-core sizes are shown to be special cases of our general approach.

Model

We consider undirected networks of N binary-valued nodes in the infinite-N limit, where the probability of a node being activated depends only on its degree k and the number m of its neighbors who are already active. Denoting this probability by $F(m,k)$ (termed the *neighborhood influence response function* (Watts and Dodds, 2007)) we require that (i) for any fixed k, $F(m,k)$ is non-decreasing with m, and (ii) once active, a node cannot become deactivated. These requirements mean that increasing the number of active neighbors of a given node will increase (or at least not decrease) the probability of the chosen node itself becoming activated. This type of positive feedback mechanism (known in the sociological literature as “positive externalities”) ensures the number of active nodes in a given realization is non-decreasing with time, and enables us to use analytical methods to calculate the mean avalanche size on locally treelike (vanishing clustering) random networks.

A broad range of dynamical problems on random undirected networks can be shown to obey conditions (i) and (ii), including Watts' model of threshold dynamics (Watts, 2002), *k*-core size calculations (Mendes *et al*, 2005), site and bond percolation problems (Callaway *et al*, 2000), and (under certain types of external field) zero-temperature random-field Ising models (Dhar *et al*, 1997). Susceptible-infected-recovered (SIR) disease transmission models may be mapped to the bond percolation problem (in steady-state) (Newman, 2002), and so are also included in the class of problems solvable using our methods.

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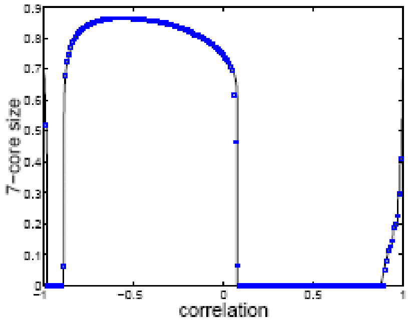

Figure 1: Theoretical prediction (curve) and numerical results (symbols) for *k*-core sizes on networks with degree-degree correlations.