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Organization of Modular Networks

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Introduction

Many real-world networks contain principally distinct modules with different architectures. In this sense they are strongly, "macroscopically" heterogeneous. For example, the Internet — the network of physically interconnected computers — is connected to mobile cellular networks. The question is how do the network's macroscopic inhomogeneity influence its global structure? What is the architecture of a modular network? How to describe modular networks using ideas previously applied to uncorrelated networks without any macroscopic inhomogeneity? We develop an approach allowing us to quantitatively describe the global organization of modular networks: to obtain the statistics of connected components in these networks and to find the distribution of intervertex distances.

The networks which we discuss are shown in Fig. 1. In a limiting case, where connections inside modules ("anti-communities" in this case) are absent, we arrive at a multipartite random network.

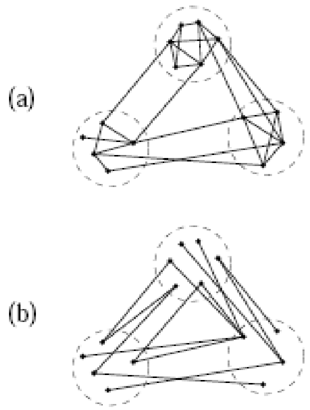

Fig.1. (a) An example of a network which we discuss in this work. The structure of interconnections between the non-overlapping modules (communities) differs from the structure of connections inside these modules. Moreover, the architectures of the modules may differ. (b) A contrasting example of a multipartite graph where connections inside the modules (extremal "anti-communities") are absent.

Results

We use a direct generalization of the configuration model of uncorrelated networks to networks with well distinguished modules. In simple terms, it is a maximally random modular networks with given distributions of inter- and intra-degrees of all modules. We show how to obtain the statistics of *n*-th components of a vertex in these network.

(The n-th component of a vertex is defined as a set of vertices which are not farther than distance n from a given vertex.) We also describe the evolution of the intervertex distance distribution with an increasing number of interlinks connecting large modules. We demonstrate that even a relatively small number of shortcuts unite the networks into one. In more precise terms, if the number of the interlinks is any finite fraction of the total number of connections, then the intervertex distance distribution approaches a delta-function peaked form, and so the infinite network is united. For more details, see Dorogovtsev *et al* (2008a,b).

Discussion

We describe a number of applications. In particular, we apply our approach to the problem of the birth of a giant connected component in the modular networks and find a significant difference with well studied macroscopically uniform networks. We discuss stransitions between various geometries in these networks and compare our results with those for more traditional nets (Dorogovtsev *et al,* 2008c). Finally, we show how our results may be generalized to modular networks with degree-degree correlations.

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