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Who wins a group-group competition ? Phase transitions in Ising models at coupled complex networks

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Introduction

We investigate the behavior of the Ising model on two coupled regular graphs and on two connected Barabasi-Albert (B-A) scale-free networks. A previous analysis based on mean field approximation has been extended and we show that a first order temperature-driven phase transition occurs in such system. The transition between antiparalelly ordered networks to paralelly ordered networks is shown to be discontinuous. For regular graphs the critical temperature is calculated from a tangency condition of a pair of equations describing weighted magnetizations of both networks. Analytical results are confirmed by numeric simulations using Monte-Carlo method. For coupled BA networks a system of coupled maps is studied and Monte-Carlo simulations are performed.

Model

In our study, we consider two interconnected regular graphs or connected B-A networks, where at each node we place an Ising spin. The interactions between the spins are ferromagnetic only. Our B-A networks are interconnected by E_{AB} links (Fig.1). Each of these links connects a node in network A with a node in network B. The nodes to be connected are chosen preferentially, i.e. the probability to pick a given node i equals $\Pi_{Ai} = k_{AAi}/\sum_j k_{AAj}$. If we perform linking in this way, the inter-network degree k_{ABi} of a node is statistically proportional its to intra-network degree k_{AAi}.

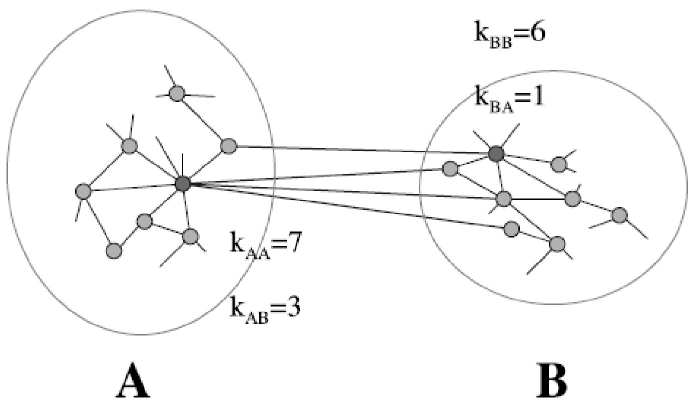

FIG. 1: Two interconnected B-A networks. A few nodes from each network are shown. The intra-network degrees k_{AA} and k_{BB} as well as inter-network degrees k_{AB} and k_{BB} for two sample nodes are presented.

The problem of the Ising model on a single B-A network has been considered in [1] and for coupled B-A networks has been studied in [2]. In connected B-A networks, Ising model is characterized by two phase transition in two different critical temperatures Tc– and Tc+. Below Tc– there are two possible phases: both networks ordered in with same spin and both networks ordered with opposite spins. At critical temperature Tc– the state with antiparallel spin ordering disappears, and between Tc– and Tc+ the system orders only parallely. At Tc+ and above the temperature is too high for network to remain ordered and it assumes paramagnetic state. As in regular Ising model, the transition at Tc+ is second order phase transition. However, unlike previous research indicated [1] the transition at Tc– turns out to be of first order. We have performed analytic calculations for a coupled system of regular graphs with constant nodes degrees , and numeric map iterations as well as Monte-Carlo simulations for coupled B-A networks.

Conclusions

We have shown that in a system of two connected networks (either regular graphs or B-A networks) one of two temperature driven phase transitions is of first order, unlike classical Ising phase transitions that are of second order. The dependence of the critical temperature on the interaction strength between the networks is complex. We have found an analytical solution for this critical point of in the case of weakly connected regular graphs. The coupled B-A networks critical temperatures are lower than results coming from the theory based in the second order phase transition. The results can be useful for illustration of competition between two social groups with opposite opinions. Numerical simulations indicate that the winner in such competition is not a group with a large number of nodes but a group with denser internal connections. The final result is dependent also on the symmetry of intergroup connections and on the number of intergroup links corresponding to the network hub.

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References

[1] A. Aleksiejuk, J.A. Hołyst, D. Stauffer, Physica A 310, 260-266 (2002).
[2] K. Suchecki, J.A. Hołyst, Phys. Rev. E 74, 011122 (2006).
[3] K. Suchecki, J.A. Hołyst, http://arxiv.org/ 0802.1499