CT 1.6.5

Voter model on Sierpinski fractals

K. Suchecki[1,2] and J.A.Hołyst[2]

[1]Virtual Knowledge Studio, Royal Netherlands Academy of Arts and Sciences, Cruquiusweg 31, 1019 AT Amsterdam, Netherlands
[2]Faculty of Physics, Center of Excellence for Complex Systems Research, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland

Introduction

The voter model is a very simple model, that may describe random migration of knowledge or opinions. It has unique properties and as such defines its own universality class that can be broadened to other similar models (Dornic 2001).
Voter model is defined by its dynamical rule. Each node has a two-state variable (we call it “spin” and assume possible values -1 and +1). During the dynamics, a node assumes state of random neighbor. This leads to dynamics that exhibit ordering depending on the dimension of the network, with D=2 being critical dimension. Below critical dimension the ordering is a power of time, while above the model does not order at all. This has been derived analytically by Frachenbourg and Krapivsky (Frachenbourg 1996).
Many systems cannot be described by regular lattices, but fractal or complex networks. The question we ask is: What are the dynamics of the voter model in non-regular fractal environment. The investigation of other models in such lattices shows that results may be quite different than for regular networks (Carmona 1998, Gefen 1980, Kohring 1986). In particular we investigate its behavior in dimensions below 2, as it approaches the critical value. As a representative of the fractal lattices we took Sierpinski Carpet.
Sierpinski Carpet is a fractal created by multiplying some basic pattern and arranging the resulting network modules in a higher hierarchy module along the same pattern (see Fig. 1 for example resulting fractal lattices). The freedom to choose the pattern allows for fractal lattices of any fractal dimension higher than 1 and not higher than dimension of the used pattern (in our case 2).

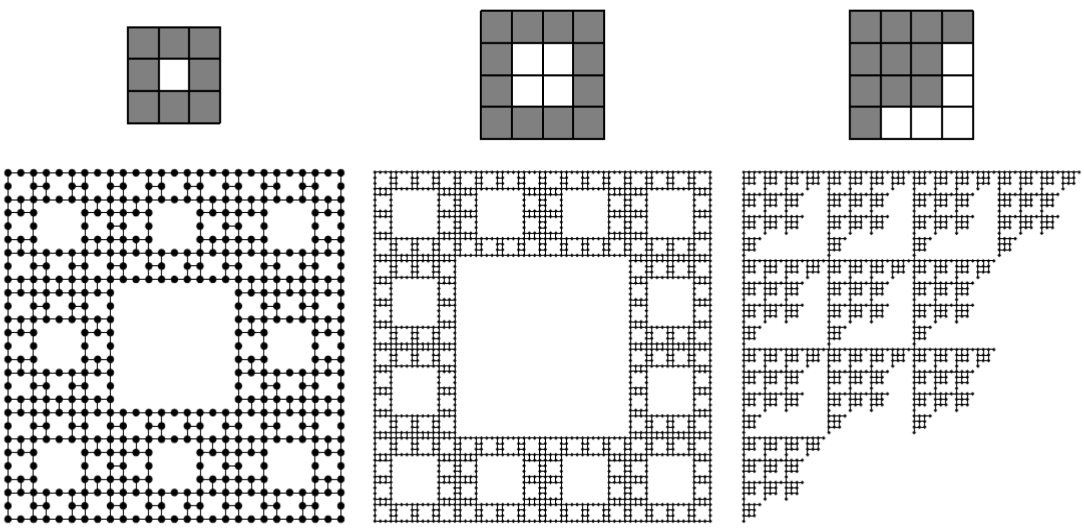

Fig 1. Few Sierpinski Carpets of different dimension and their patterns.

Results

Numeric investigation of the voter model dynamics on Sierpinski fractals was performed using computer simulation (Suchecki 2006). The results show that the ramification of the fractal plays crucial role in the dynamics.

For infinite ramification (fractals without “weak points”, like first two on Fig.1) the ordering is analogous to regular lattices with integer dimension. For finite ramification (fractals with “weak points”, like the right one on Fig.1) the resulting ordering process exhibits a log-periodic oscillations.

We give heuristic explanation of these oscillations (see Fig.2), additionally noting that log-periodicity is a mark of discrete scaling – a property that all deterministic fractals share.

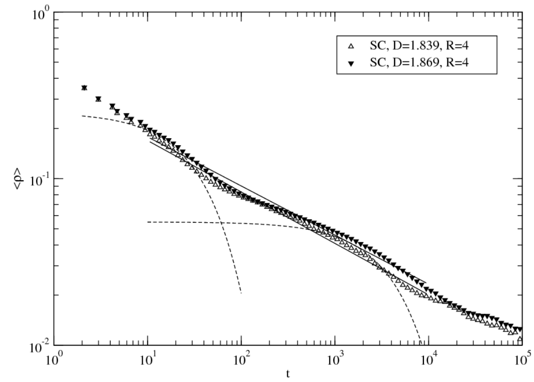

Fig 2. Inter-domain surface for ordering of voter model in finitely ramified Sierpinski Carpet lattices

Conclusions

It has been shown that voter model dynamics depend mainly on dimensionality, but in case of fractal lattices also on their ramification. The analytic calculations used for regular lattices give qualitatively correct results, although quantitative agreement is not found.

Acknowledgement

This work was partially supported by a EU Grant Measuring and Modelling Complex Networks Across Domains (MMCOMNET) and by State Committee for Scientific Research in Poland (Grant no. 1P03B04727).

References

J.M. Carmona, U.M.B. Marconi, J.J. Ruiz-Lorenzo, A. Tarancón (1998): Phase transitions on Sierpinski fractals. Physical Review B 58, 14387.

I. Dornic, H. Chaté , J. Chave, H. Hinrichsen (2001): Critical Coarsening without Surface Tension: The Universality Class of the Voter Model. Physical Review Letters 87, 045701.

L. Frachenbourg, P.L. Krapivsky (1996): Exact results for kinetics of catalytic reactions. Physical Review E 53, R3009-R3012.

Y. Gefen, B.B. Mandelbrot, A. Aharony (1980): Critical phenomena on fractal lattices. Physical Review Letters 45, 855.

G. Kohring (1986): Fractal lattices and universality. Phyical Review B 33, 610.

K. Suchecki, J.A. Hołyst (2006): Voter model on Sierpinski fractals. Physica A 362 (2): 338-344