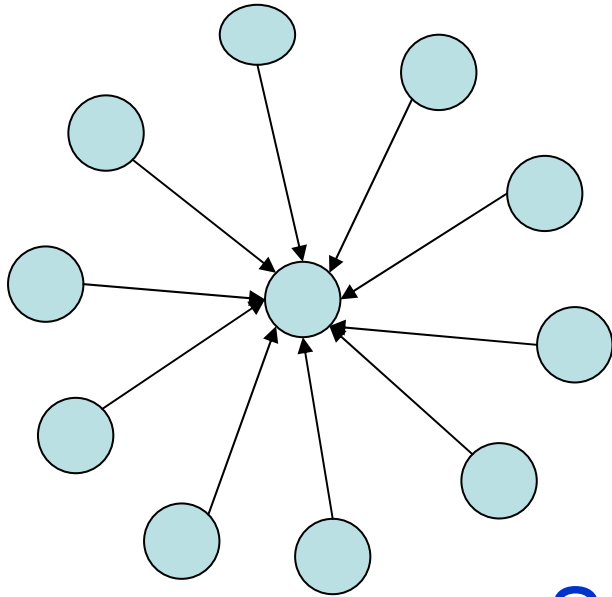


Economics of Networks

Introduction



Sanjeev Goyal
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Networks in economics

- Networks come in different guises:
 - primarily social. e.g. friendships, family, sport clubs.
 - primarily economic: research collaboration networks between firms, trading networks, inter-bank networks.
 - combine elements of the two. Co-author networks, world wide web, colleagues.

Networks in economics

Traditionally, social structures were viewed as important in backward economies: with industrialization and modernization, this role gradually declines and eventually disappears.

Similarly, trading restrictions were viewed as a departure from perfectly functioning markets where everyone can trade with everyone else on common prices.

The most well known exposition of this argument is *The Great Transformation* by Karl Polanyi.

Networks in economics

Until the 1970's economics was based on an acceptance of this view.

Economists took the view that economic activity takes place between

- anonymous agents
- who meet in centralized markets
- and trade at common prices.

General equilibrium theory, oligopoly theory, the theory of search and matching frictions, epitomize this approach.

- This theory yields deep insights and has been very successful.

Networks in economics

- The 1970's asymmetric information revolution: economic exchange take place in the shadow of imperfect and asymmetric information.
 - Buyers do not know quality of products they buy, and the seller knows more.
 - Banks do not know the quality of project, while entrepreneur knows more.
 - Managers do not know the effort which workers exert, and must reward them based on imperfect signals such as output.
 - Economic growth takes place due to technological change. Firms, farmers and consumers adopt new products under imperfect information of their profitability.
- ***Institutions – formal and informal -- play a key role in mitigating these information problems.***

Networks in economics

- Interaction between individuals is very restricted: we meet a very small subset of the population. The descriptive plausibility of these local interactions was not contested but its significance was not examined either.
- 1990's research: showed that local interaction yields very different predictions as compared to centralized interaction (as in general equilibrium and oligopoly interaction) and in random interaction (as in search theory).

Networks in economics

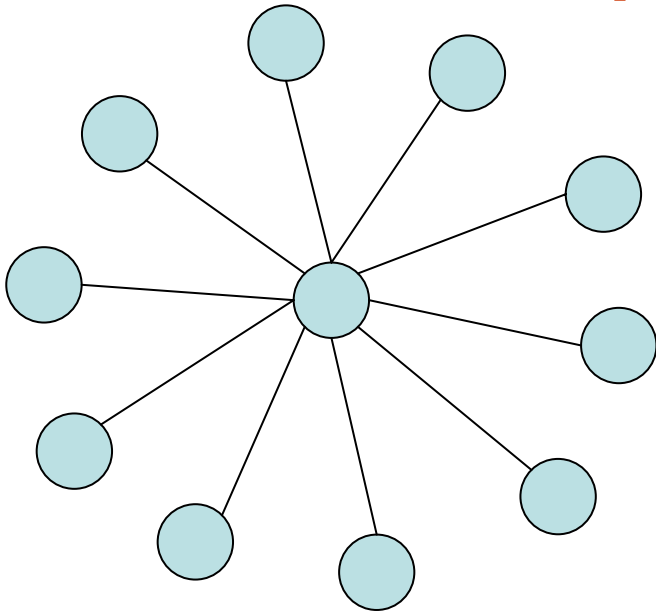
- The realization that informal institutions are very important along with research on local interaction led to a revolution:
- Two key elements:
 1. individual identity is important.
 2. patterns of interaction are crucial.
- These two elements have fuelled research on networks in economics, which I will talk to you about today.
- In my talks, I will:
 - start with the formation of networks [\[Lecture 1\]](#)
 - then the consequences of networks [\[Lecture 2\]](#)

Economics readings

- Sanjeev Goyal (2007) *Connections, an introduction to the economics of networks*. Princeton University Press.
- Matthew Jackson, (2008), *Social and economic networks*. Princeton University Press.
- Fernando Vega-Redondo (2007), *Complex Social Networks*. Cambridge University Press.

Economics of networks

Strategic Network Formation



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Outline

1. Structure of networks
2. Strategic foundations of networks
3. Theory 1: Communication networks
 - Stars, hubs and spokes
 - Networks Advantages
 - Good and bad networks
4. Theory 2: Networks and markets
 - Collaborating to compete
 - Dominant groups
 - Subsidizing collaboration to build dominance.
5. Main insights and open problems

1. The structure of networks

- A network describes a collection of nodes and the links between them.
- Once you begin to study networks it is difficult not to see them everywhere.
- Examples include: Internet, World wide web, airline networks, friendships, research alliances, co-authorships, trade and exchange, *guanxi*.

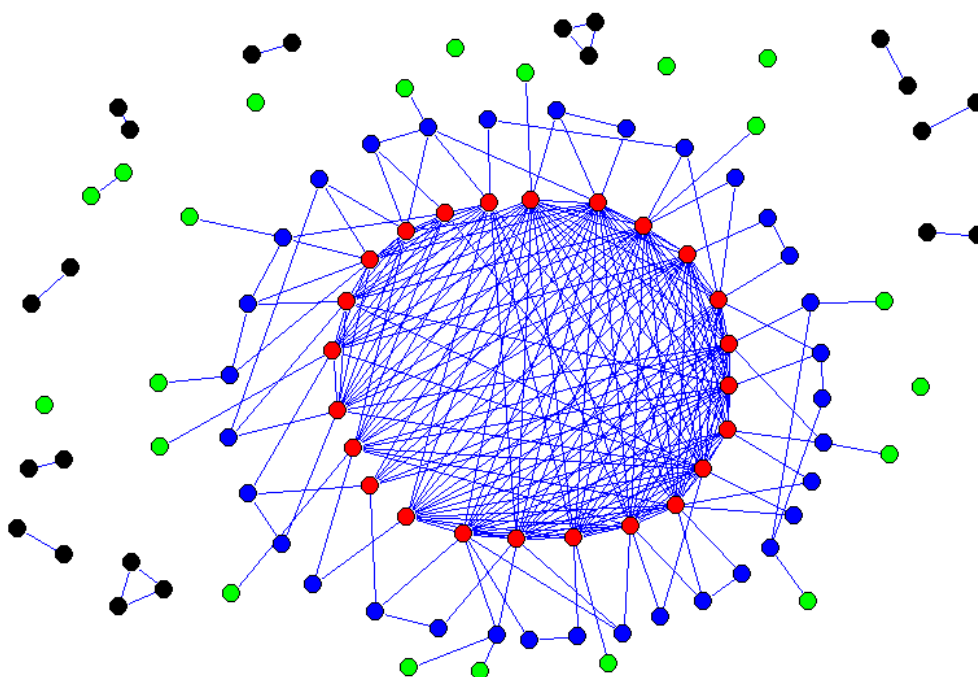


Figure 2.10: Research collaboration among firms

1. Structure of networks

- Three key properties: the **degree distribution**, the **clustering**, the **average distance**.
- **Degree distribution:** *Average degree is very small compared to number of nodes and there is enormous inequality.* On the web, in a study from 2000, there were over 200 million web sites, average degree only 7.5, most web sites less than 10 links, but some have thousands of links!
- **Clustering:** *Clustering is very high in social networks.* If links are formed at random then in a large n -node network with average degree k the clustering would be roughly k/n . In the economics co-author network the clustering coefficient was 0.157; over 7,000 times the level in a random network.
- **Average distances:** *The average distance between nodes is very small.* In the world wide web, the giant component comprised about 180 million web sites and the average distance was only 6. Similarly, in the firm network there are over 4000 nodes, but the average distance was around 4.

1. Structure of networks

- **Small worlds:** A network with small average degrees, high clustering, and small average distances has been called a *small world* network by Watts and Strogatz (1998). The expression is originally due to Milgram (1967).
- Social and economic networks display common features: low average degree, very unequal degree distribution, clustering is high and the average distance between nodes is small. Thus star network and its variants -- such as inter-linked stars and core-periphery networks -- capture essential elements of empirical networks.
- **Key questions:** Who forms the networks? When do they have this structure? Why does it matter?

2. Strategic foundations of networks

Key features of linking activity

- 1. Linking is a decision:** Individuals choose on forming links.
Examples: Scientists decide on whether or not to collaborate
Firms can choose whether or not to form an alliance;
I decide whether or not to form a hyperlink with your homepage.
 - 2. Externality/spillover:** A link between 1 and 2 affects the payoffs of 3 as well as the rewards to 3 from linking with 1 and 2.
Examples: capacity constraints in scientific collaboration;
firm 1 and 2 collaborate affects firm 3 in same market.
- Combine 1 & 2: **Games of Network Formation.**

2. Strategic foundations of networks

- Key issues in modelling:
 1. Payoffs: linking generates rewards and entails costs.
We define these formally.
 2. Power: who decides on the link, one person, two persons, all players etc.
 3. Information: what do I know -- about other players and about the network -- when I form a link?

We start with the simplest case: a player decides on whether to link with others unilaterally. No transfers or bargaining. Full Information about rewards and costs of linking and about the network.

3. Theory 1: communication networks

- **Players:** Large number, n .
- **Strategy:** Link with any subset of others. A link is 0-1.
- **Payoffs:** Each link is costly and link between 1 and 2 gives 1 access to information which 2 has on her own, **AND** information which she accesses via her links.

Thus: payoff is *increasing* in the number of people she accesses directly or indirectly and *decreasing* in the costs of links she forms.

Example: Payoff = [#people accessed] Value per person
– [cost link]. [# links formed]

- **What is the network that emerges from this process?**

3. Communication networks

- How do we solve this game?
- ***Nash equilibrium:*** This is a profile of strategies one for each player with the following property: *every player is doing as well as possible, given what others are doing.*
- What is a Nash equilibrium of the game of network formation?

3. Communication networks

Some simple intuitions

1. Suppose $C > (n-1)V$: then no linking always.

The empty network is only possibility.

2. Suppose $C < V$: then 1 wants to access everyone else irrespective of what others do.

Every player accesses everyone else; connected network is only possible outcome.

3. Suppose $V < C < (n-1)V$ then linking depends on other's linking behaviour:

If no one links, it is optimal to form no links: empty network is an equilibrium.

If people form links then it may be optimal to connected with them. This is the interesting case

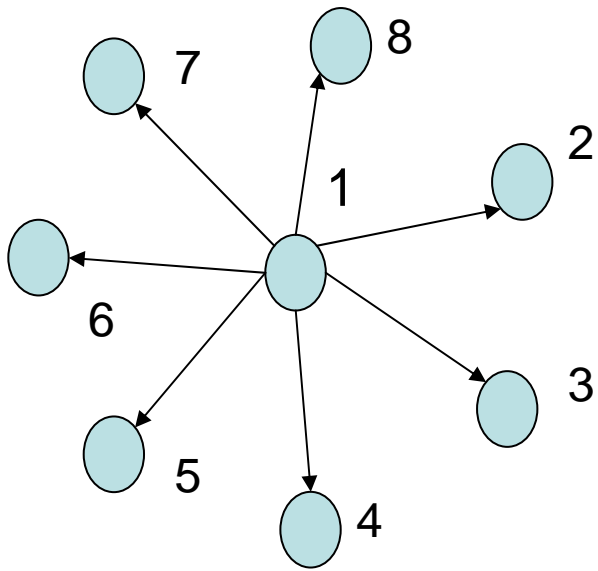
3. Communication networks

- **Theorem 1**

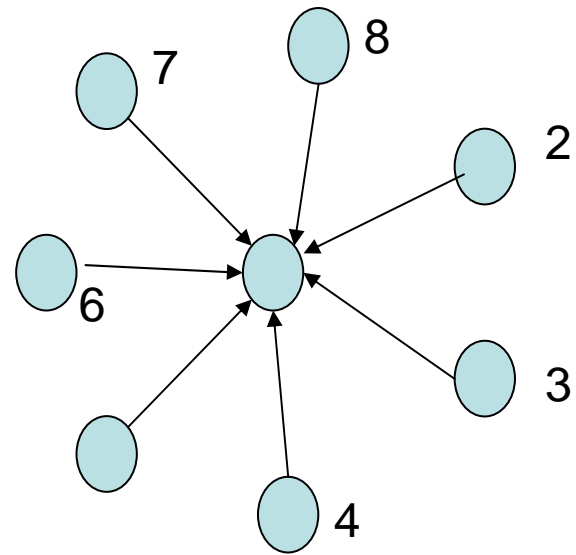
The star is the unique equilibrium network architecture.
[Bala and Goyal, 2000. Hojman and Szeidl, 2007, Ferri, 2007].

- Key intuitions:

1. *The star is an equilibrium:* the spokes are accessing everyone with just one link, and everyone is close by.
2. *Why is nothing else equilibrium?* **Networks must exhibit small world property.** Take any two end-players in a `tree' network. They have an incentive to get closer to the centre, if long paths are bad for information transmission.



Centre sponsored star



Periphery sponsored star

Equilibrium networks

3. Communication networks

- Dynamics: suppose players can observe the network and revise links over time. So network will evolve over time.
Question: starting from an arbitrary network, will the dynamics converge, and if yes, what is the long run network?
- **Theorem 2**
Starting from any network, the dynamics converge to the star network. [Bala and Goyal, 2000; Ferri 2007].

General message: unequal degrees and short average distances arise in equilibrium.

3. Communication networks: advantages of networking

- Does network degree and location confer advantages?
- Network formation leads to star in which the central hub player has privileged access to information. In general, the spokes pay for the links, and so hub gains both ways.

General message: *strategic networking can create large inequalities across players who are ex-ante identical.*

3. Communication networks: good and bad networks

- Key idea: links are motivated by individual incentives. Individual linking generates externalities and spillovers on others. So there is a tension between equilibrium and socially desirable networks.
- In communication network game: individual linking creates *positive* spillovers for others, and so individuals typically form less links than is socially desirable.

General message: *Equilibrium networks are under-connected, relative to what is socially desirable.*

4. Networks and Markets

Leading firms in hi-tech industries rely on a combination of in-house and collaborative research to stay competitive in markets.

E.g., Bayer, Merck, Pfizer etc. IT alliances

Key ideas:

- Strategic alliances in the face of competition.
 - an alliance improves competitive position of partners
 - it also alters incentives of other firms to initiate costly new alliances.

4. Networks and markets

Firms bilaterally choose research links.

- Partners share technological information which lower costs of production. More links firms lead to lower costs, which leads to larger market share.
- However each link involves a fixed cost C .

Key features: 1. link decided bilaterally.

2. alliances arise in response to market pressures and they in turn define competition in networks.

4. Networks and markets

A Game of Network Formation:

- Players: There are n firms.
- Strategies: Each firm announces intention to form 0-1 links with others. A link is formed between 1 and 2 if both 1 and 2 announce an intention to form a link.
- Payoffs: Links which are formed cost F to each firm and lower their costs of production by c . Links formed define a network, which defines a vector of firm costs.

The gains from links depend on the nature of market competition. We explore this key relation between market conditions and incentives to form alliances.

Strong competition: firm makes profits only if it is unique lowest cost.

Moderate competition: lower costs imply higher profits.

4. Networks and markets

- How do we solve games with bilateral link formation?
- Nash equilibrium is too permissive, as there is a coordination problem in bilateral link formation, firm 1 offers no link since it expects no one else to form any links.
- To avoid this problem: refine Nash equilibrium with the requirement that no two players should have an incentive to form an additional link.
- Leads to ***pairwise equilibrium***

4. Networks and markets

- **Theorem 3:**

With strong competition, the empty network is the unique pairwise equilibrium. [Goyal and Joshi (2003)]

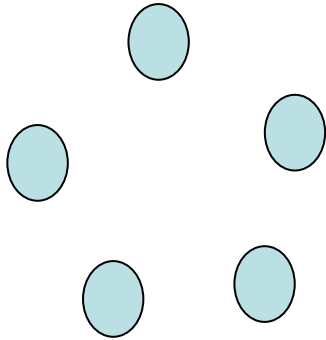
Intuition: in non-empty network, there is always a firm which forms link but makes no profits. Hence, cannot recover costs of link. Better to delete all links!

- **Theorem 4:**

With moderate competition and small costs of forming links, the complete network is the unique pairwise equilibrium. [Goyal and Joshi (2003)]

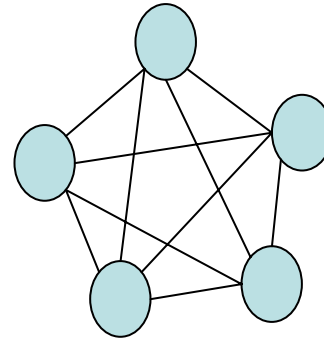
Intuition: when two firms form links, they gain at the expense of other firms, and this covers costs of links. Always form links.

General Message: market competition crucial in defining networks



Empty network

Strong competition



Complete network

Moderate competition

**Equilibrium networks under
different market conditions**

4. Networks and markets

- We now turn to the case of high costs of forming links.

FIRST NOTE: Result for strong competition unaffected.

But moderate competition now becomes very interesting:

- Key issue: additional link creates a cost of F : so link formation depends on how rewards from additional links are affected by # own and LINKS and others links:
 - Whether marginal returns are increasing/decreasing in own links?
 - Whether linking by others increases or decreases my returns?

Key property of the moderate model: **marginal payoffs are increasing in own links and decreasing in the links of other firms.**

- What are the pairwise nash equilibrium networks?

4. Networks and markets

- **Theorem 5**

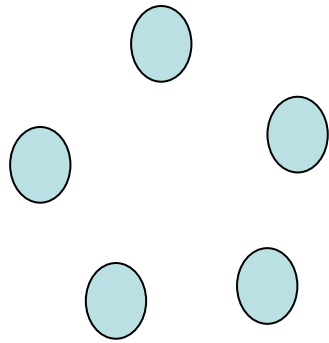
Under moderate competition with significant costs of forming links, pairwise equilibrium network has the dominant group architecture. [Goyal and Joshi (2003), Deorian (2006)]

Dominant group network contains a completely linked group of firms, and the rest of firms are isolated.

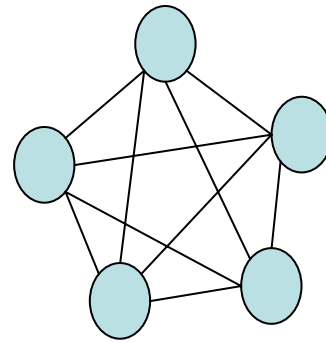
Intuition: Two or more groups of connected firms not possible, due to linear costs of links and convex returns from own links. Note that firm with connections always wants more links, but isolated firm may not link since its marginal returns are too low.

- **Theorem 6**

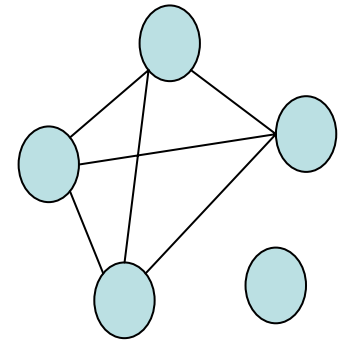
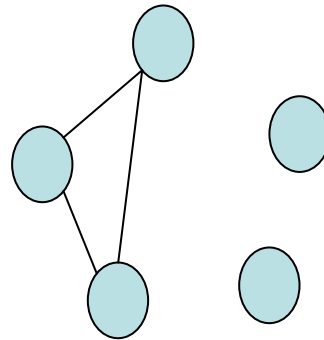
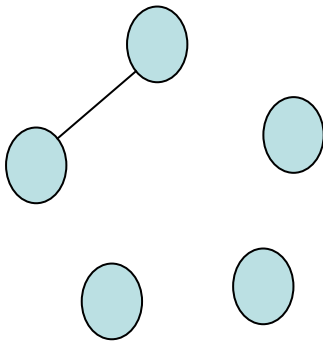
The firms in the dominant group earn strictly more profits than the isolated firms.



Empty network

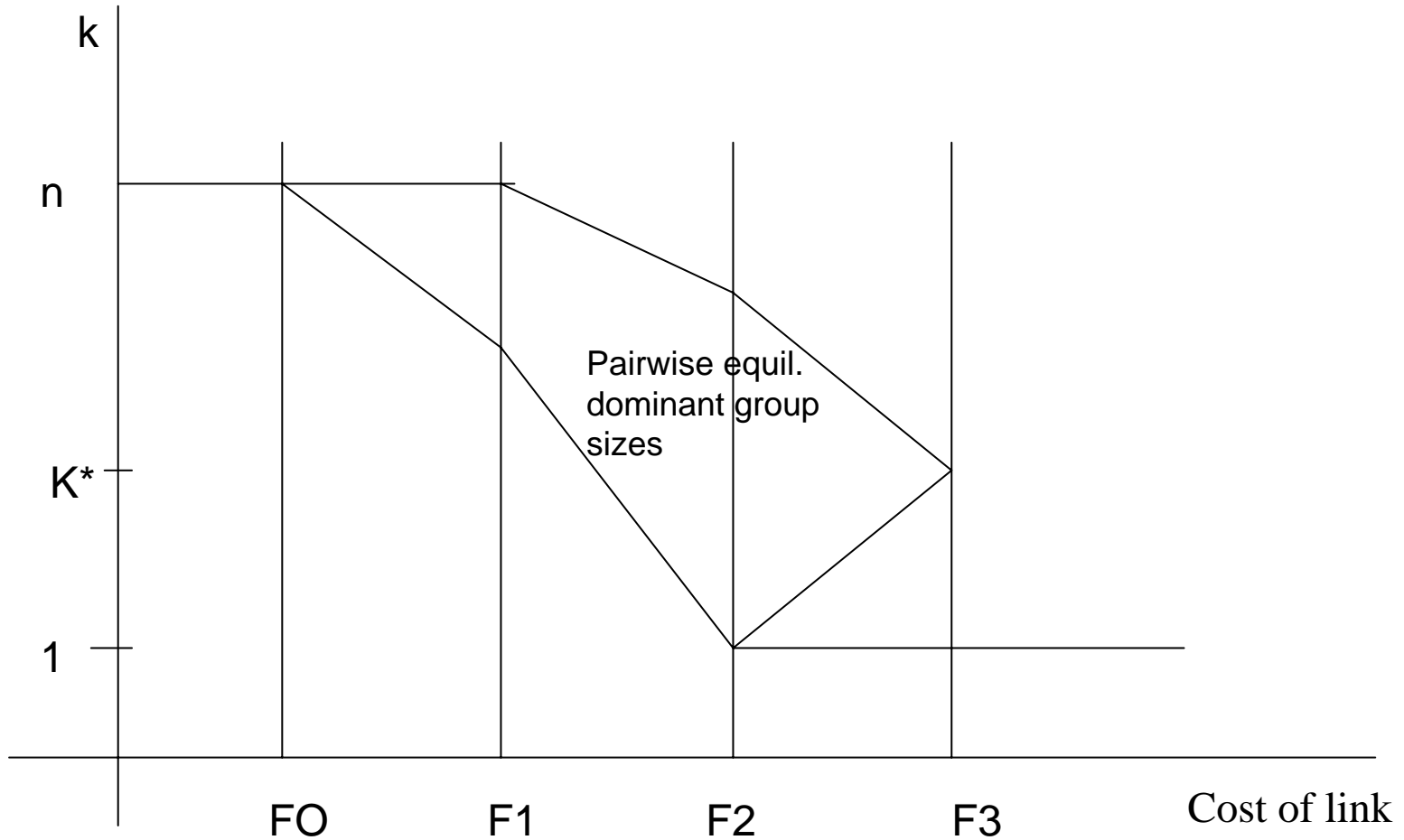


Complete network



Dominant group networks

Size of dominant group



Size of dominant group vs costs of linking

4. Networks and markets

- **Theorem 7**

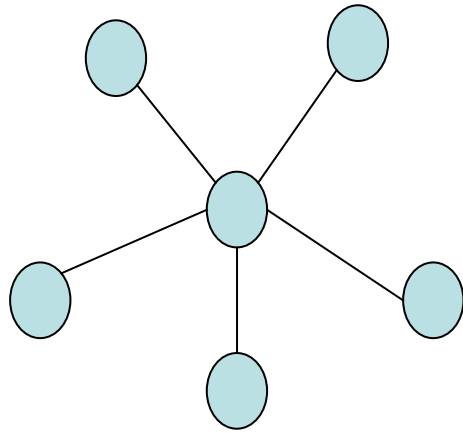
If firms can make transfers to other firms in forming links, then a star network and related structures are pairwise equilibrium networks.[Goyal and Joshi, 2003]

Intuition: Marginal gains from links are increasing in # of own links, so the central firm in a star has high marginal returns from new link. Its many connections lower the returns to peripheral firms, who wish to form no links. So central firm subsidizes links and peripheral firms form no links among themselves!

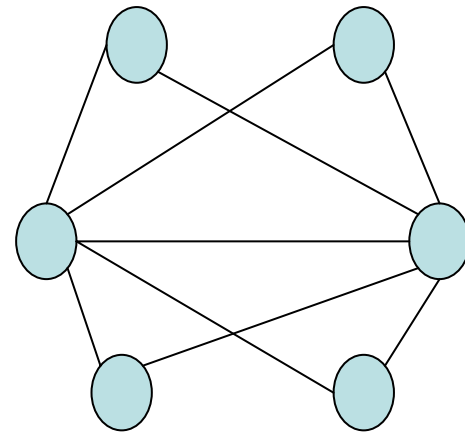
- **Theorem 8**

The central firm with more links earns larger profits than the peripheral firms with few links. [Goyal and Joshi, 2003]

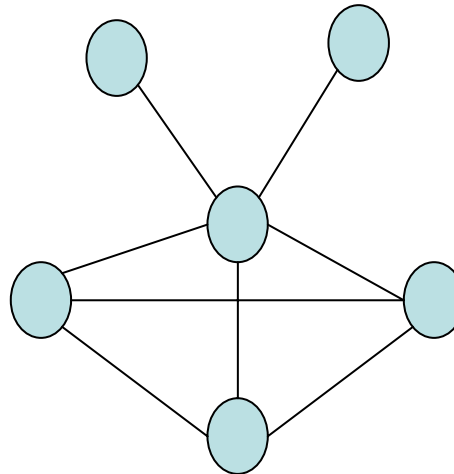
Key idea: Subsidizing links to create market advantages....



A. Star Network



B. Inter-linked star (2 centres)



C. Generalized inter-linked star

Equilibrium networks with transfers

4. Networks and markets

- General Message:** strategic networking by firms gives rise to
- unequal degrees,
 - small average distance,
 - high clustering.
 - firms with many connections actually subsidize formation of links to create larger profits [network advantages].

4. Networks and markets: socially desirable networks?

- In the firms research networks: link between 1 and 2 actually lowers profit effects of other firms; so there is a negative externality created by links: *firms create too many links in equilibrium*. [Goyal and Joshi 2003. Yi, 1998]
- Policy: Governments all over the world try and facilitate inter-firm collaborations; even subsidize them. Our results raise doubts about the optimality of such a policy.

5. Summary: strategic network formation

- The theory of network formation is concerned with understanding how networks arise out of strategic choices of players concerning link formation.
- The theory generates surprisingly sharp predictions on equilibrium networks: unequal degrees, small average distance, arise naturally. Good match with empirics.
- Strategic networking has powerful effects on payoff inequality as well as aggregate social welfare.
- Suggests role of policy – taxes and subsidies – to reorient network formation. [Taken up by the theory of mechanism design.]

5. Current research & open problems

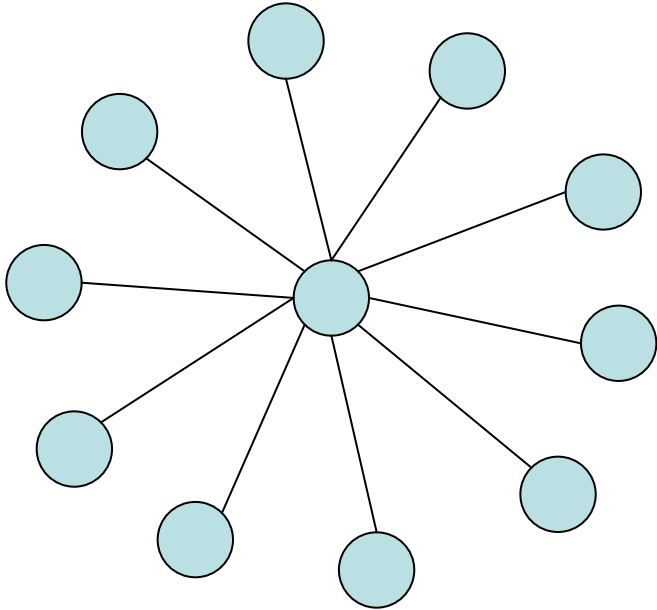
1. Weighted graphs: Bloch & Dutta, 2008; Goyal 2005; Bruckner 2005.
2. Applications: trade, labour, crime, industry, education: Calvo & Zenou 2005; Goyal & Joshi 2006; Belleflamme & Bloch 2005.
3. Combining linking and search for jobs and prices. Galeotti 2008.
4. Linking activity and coordination and cooperation games: Jackson & Watts 2002; Goyal & Vega-Redondo, 2006; Vega-Redondo 2006.
5. Networks and markets: social networks & markets shape exchange production & consumption Kranton & Minehart 2001.
6. Pure theory of network formation: decision power, bargaining, auctions in networks. Jackson 2006; Bloch & Jackson 2006; Kranton & Minehart 2001.
7. Strategic network design: designer faced with adversary, Baccara and Bar-Isaac 2008; Goyal and Viger 2008.

5. Open problems

- A. Dynamic network formation: network advantages suggest the pressure to pre-empt others in the creation of links. Very important open problem!
- B. Network formation with large number of players: key role of incomplete information about players and about networks.
- C. Networks and markets: traditionally economists focused on markets and ignored social structures; recent work focuses on networks and ignores markets. There is an urgent need for research and models which integrate networks and markets. This is also key aspect of developing economies, as networks evolve and give way to markets in such economies.

Economics of networks

Network effects



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Social influencers & wisdom of crowds

- Individuals choose among actions whose payoffs are unknown.
- They use own and others' past experience as a guide. Choices of close by others specially influential due to higher reliability, similar preferences, lower access costs.
- Leading examples:
 1. Consumer choice: which brands, products, designs to buy?
 2. Medical innovation: which medicine to prescribe?
 3. Agricultural practices: crops and input combination choice.
 4. Health practices: fertility treatments, alternative medicine?

Social influencers & wisdom of crowds

- General framework:
 1. Individuals located in a social network of communication.
 2. They choose between actions without knowing the rewards.
 3. They use own past experience, experience of friends & neighbors.
 4. Choose an action that maximizes individual payoffs.
- Our objective: Study how network of communication affects the actions, beliefs and payoffs over time.

Social influencers & wisdom of crowds

- Some aspects of empirical communication networks we should keep in mind:
- Very asymmetric structure: a few people observed by most others, and most people not observed by anyone!
- Examples: www, rural communities, informal networks in firms, social networking sites.
- We would like to understand the affect of this asymmetry.

Social influencers & wisdom of crowds

- Time: $t=1,2,\dots$ and agents: $N = 1,2,3,\dots,n$.
- At each point in time t , each individual chooses between two actions A and B.
- There are two states of the world 1 and 2.
 - In state 1, action A yields 1 with prob. 0.75, and 0 with prob. 0.25,
 - In state 2, action A yields a payoff of 1 with prob. 0.25, and 0 with prob. 0.75.
 - Action B always yields 1 with prob. 1/2.
- So action A is optimal in state 1 and action B is optimal in state 2.
- Individuals DO NOT know the true state: Let p be belief that true state is 1. Then A is optimal if p is greater than $\frac{1}{2}$ and B otherwise.

Social influencers & wisdom of crowds

- **Network of communication:** Each individual observes the outcome of his own action, and the outcomes of actions of his neighbors. Let $N(i,g)$ be the set of observed agents.
- **Belief Dynamics:** In every period, t , person chooses an action $a(i,t)$ optimal with respect to beliefs $p(i,t)$. She observes actions and outcomes in neighborhood and updates beliefs using Bayes' Rule.
- **AIM:** We study the influence of network g on actions, beliefs and payoffs.

Social influencers & wisdom of crowds

- Individual actions are an optimal response to beliefs, which in turn evolve in response to the information generated by actions. Thus the dynamics of actions and beliefs feed back on each other.
- An individual observes the outcomes of own actions and the actions and outcomes of neighbors, so his beliefs will evolve depending on the particularities of his experience.
- As time goes by, and his experience grows, additional information should have a smaller and smaller effect on his views of the world. This intuition leads to:
- Theorem 1: *The beliefs and utilities of every individual converge in the long run.*[Bala and Goyal, 1998]

Social influencers & wisdom of crowds

- We now relate networks to long run payoffs.

Theorem 2: *Every individual in a connected society gets the same long run utility.* [Bala and Goyal 1998]

- Intuition: If I observes J then she must be able to do as well as j in the long run. Iterate and note that this is true for K who observes J via I, indirectly. The proof follows since I observes J for every pair I and J in a connected society.

Social influencers & wisdom of crowds

- Key question: Are limit actions and utilities optimal and how does social communication matter?

Example: *Social influencers and herding*

- Suppose true state is 1, and everyone is optimistic, so $\frac{1}{2} < p(i,1) < p < 1$. Everyone chooses A in period 1.
- Now suppose that individuals are located on line network and their observation window is: $\{i-1, i, i+1\} + S$, where S are observed by everyone, i.e., they are the ***social influencers***.

Social influencers & wisdom of crowds

Proposition: *Suppose true state is 1, and optimal action is A. Assume everyone is optimistic. There is a strictly positive probability that everyone chooses sub-optimal action B after period 2, irrespective of size of society!*

- Notice that for large n plenty of information is generated in favor of action A. Yet, this information is overwhelmed by the negative information of a few individuals, S , the social influencers!
- Argument: Suppose that S all get 0 in period 1. This happens with probability $(0.25)^S$. Any individual i gets a minimum residual of $S-3$ negative signals. For large enough S , this means $p(i,2) < 1/2$, for all i . Everyone chooses action B in period 2;
- No information is generated by B, everyone chooses B ever after.

Social influencers & wisdom of crowds

Example: Society without influencers S:

Suppose $N(i,g)=\{i-1,i,i+1\}$, for everyone.

Proposition 2: In this society with pure local interaction eventually everyone chooses action A, if society is large enough.

Then since A is optimal, there is positive prob. Q that $i,i-1,i+1$ all receive positive information about A. Thus prob. of i choosing B is bounded above by $1-Q$. Similarly, for individual $i+3$.

- Since i and $i+3$ do not share any neighbors, prob that neither i nor $i+3$ tries the optimal action is bounded above by $(1-Q)^2$. Similarly prob. that m individuals with no common neighbors only try B is bounded above by $(1-Q)^m$. This converges to 0 as m gets large.

Social influencers & wisdom of crowds

- Two individuals i and j are ***locally independent*** if they do not observe anyone in common.
- Theorem 3: *In a connected society probability of everyone choosing A goes to one, as we increase locally independent individuals.* [Bala and Goyal 1998]
- Remarks:
 1. Connections help convey valuable information, while local independence preserves space for individual experimentation and prevents premature lock into sub-optimal action.
 2. Absence of social influencers good for long run outcome.

Social influencers & wisdom of crowds

- We would like now to define social influence more precisely and study how this influence is related to the process of learning.
- Suppose that everyone starts at date 1 with a belief $p(i,1)$. During period 1 everyone updates his belief: taking average of the beliefs of neighbors and his own belief. She assigns weight $w(i,j)$ to agent j .
- Thus belief dynamics:

$$p(i,t+1) = w(i,1)p(1,t) + w(i,2)p(2,t) + \dots + w(i,n)p(n,t).$$

- Define beliefs at time t as $p(t) = \{p(1,t), p(2,t), \dots, p(n,t)\}$.
- Let W be the matrix of weights of influence.
- **Our aim: *study the evolution of beliefs in relation to the matrix W .***

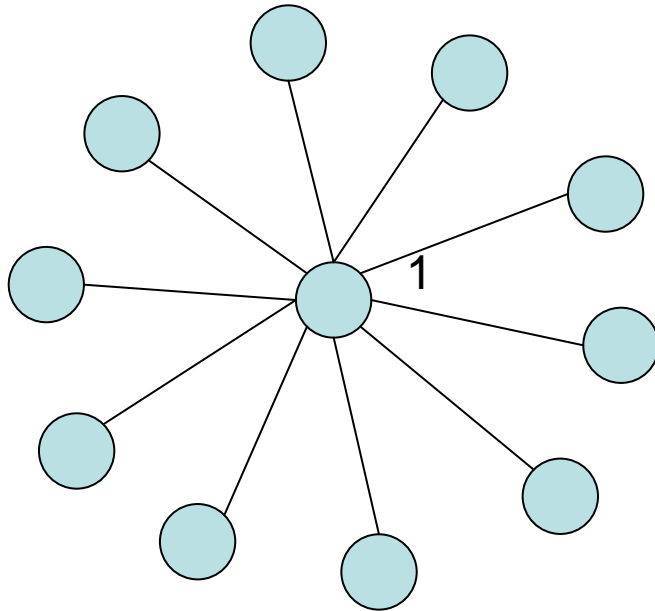
Social influencers & wisdom of crowds

- Consider a connected society in which people do not disregard their own opinions, i.e., $w(i,i) > 0$, for everyone. The social influence matrix W is then a stochastic, aperiodic and irreducible matrix. The following nice result on beliefs and social influence obtains:
- Theorem 4:
Suppose that the matrix W comes from a connected society and that $w(i,i) > 0$ for all i . Then
 1. *The influence of j on i converges: $\lim w^t(i,j) = w(j)$.*
 2. *The social influence vector is w , where $w^T W = w^T$.*
 3. *The beliefs converge for everyone, $p^* = w^T p(i, 1)$.*

[DeMarzo, Vyanos, Zweibel, 2003]

Social influencers & wisdom of crowds

- Example: social influence and beliefs
- Suppose the network is a star:



Social influencers & wisdom of crowds

Summary

- Main question: ***How does the network structure affect individual actions and utilities over time?***
- In a connected society with Bayesian learning
 - Beliefs and utilities of all agents are equal eventually.
 - Key role of social influencers: local independence sufficient for learning the truth and choosing optimal action.
- In a connected society with naive learning:
 - A precise measure of social influence relating to eigen vectors of social network matrix.
 - Individual social influence must shrink to 0 for wisdom of crowds to prevail.

Who's who in networks: Finding key player

- This part of the lecture will illustrate how the theory of games on networks can be used to devise policy.
- We will first lay out a number of examples to illustrate the scope of such games, and then we will present some simple but powerful results on how network structure affects individual behavior.

Example : Local public goods

- Individuals exert costly search effort to get price quotes from different firms.
- They share the information they gather with their friends, neighbors and colleagues.
- They then make purchase decisions.
- The benefits of search depends on own efforts and the search efforts of one's neighbors.

Questions:

- Do more connected people collect less information and free ride on others?
- Does a more connected society make more or less informed decisions?

R&D Networks

- Firms increasingly choose to collaborate in R&D with a view to lowering costs of production as well as improving product quality.
- Two aspects of collaboration are prominent.
 1. the relationships are non-exclusive.
 2. firms often collaborate with other firms within the same market, giving rise co-opetition.

Questions:

- One, do better connected firms exert higher effort and earn larger payoffs as compared to poorly connected firms,
- Two, does an increase in collaboration benefit the firms?

[Goyal and Moraga 2001, Belleflamme and Bloch 2005]

Social networks and criminal activity

- There is considerable empirical evidence on the existence of interaction effects on levels of criminal activity.
- The basic method of establishing such effects is to show that variations in criminal activity spatially or across time cannot be explained in terms of differences in economic and social indicators. The residual variation is attributed to social interactions.
- The key observation: complementarity between criminal activity of individuals creates the possibility of multiple social outcomes.

Social networks and Crime

- There is an overall value which can be shared among the criminals, and an increase in efforts by individual i , leads to higher share.
- On the other hand, individuals learn about crime through social interaction and so increased criminal activity by friends helps an individual.

- Suppose that payoffs to player are given by

$$R(s) = a \cdot s_i + \frac{1}{2} b s_i^2 + c [w(i,1) s_i \cdot s_1 + w(i,2) s_i \cdot s_2 \dots].$$

- Assume that $a > 0$, while $b < 0$ and $c > 0$, reflects the effects of neighbors on player i 's payoffs.

Bonacich centrality

The basic idea is that the network centrality of a player reflects the number of paths of all lengths between this player and everyone else.

Consider the adjacency matrix G of network g , and players i and j are directly connected if and only if there is a direct link between them. Let G^k be the k^{th} power of the matrix. What is the interpretation of this matrix? The entry $g_{i,j}^k$ measures the number of paths of length k that exist between players i and j in network g . Examples?

Let $a \geq 0$ be a scalar. Define the matrix $M(g, a)$ as follows:

$$M(g, a) = [I - aG]^{-1} = \sum_{k=0}^{\infty} a^k G^k. \quad (1)$$

Definition 1 *Bonacich centrality of players given parameter c is $b(g, c) = [I - cG]^{-1}$. Bonacich centrality of player i is then defined as follows:*

$$b_i(g, c) = \sum_{j=1}^n m_{i,j}(g, c). \quad (2)$$

This counts the total number of paths of all lengths (suitably weighted) starting from i in network g .

Individual behavior and Bonacich centrality

The following result characterizes individual efforts as a function of the network.

Proposition 1 *There is a unique equilibrium $s^* = \{s_1^*, s_2^*, \dots, s_n^*\}$, which is characterized by*

$$s^* = \frac{a}{b}b(g, c/b). \quad (3)$$

[Ballester, Calvo-Armengol and Zenou, 2006].

Remarks: 1. Equilibrium effort directly proportional to network centrality, and no other feature of the network. 2. Only one type of network centrality matters, Bonacich centrality. 3. Adding links raises the effort of everyone, as centrality goes up!

Interesting question: Who is the key player to remove if we wish to minimize crime? Suppose a public agency can remove a node: which node should it remove from the network to maximize crime reduction? Is the most central player also the key player?

It turns out that the key player is the one who has most effect on others, while the Bonacich centrality measures paths from a node to all nodes including itself. To have maximum impact we want to take the most central node who has most impact on others, so we want to discount paths leading back to the node. So The *key player* is given by:

$$K(g) = \frac{b_i(g)^2}{m_{ii}(g)}. \quad (4)$$

Who's who in networks: Finding key player

Summary

- Empirical work suggests that social networks important for criminal activity: We seek to understand how network structure maps into individual behaviour.
- Main findings:
 1. Equilibrium actions proportional to Bonacich centrality of an agent! Thus only one feature -- centrality -- and only one type of centrality -- Bonacich centrality -- matters for equilibrium actions.
 2. Key player: Removal of players to maximize effect. Player with highest inter-centrality.

Current research and open problems

- **Combining networks and markets:** Bloch and Querou (2008), Ghiglino and Goyal (2008).
- **Games and statistical networks:** empirical networks complex; so players have limited knowledge of the network. Study how different types of knowledge of networks affects behavior. Galeotti, Goyal, Jackson, Vega-Redondo and Yariv (2007).
- **Influencing the influencers:** how can firms use knowledge of networks to optimally influence social influencers? What is the value of network information for firms? Galeotti and Goyal (2007).
- **Risk-sharing in networks:** Bloch, Genicot and Ray (2007).
- **Trust and social capital in networks:** Mobius and Szeidl (2008).

References

- V. Bala and S. Goyal (1998), Learning from neighbors,
- C. Ballester, A. Calvo-Armegol and Y. Zenou (2006), Finding the key player: who's who in networks?
- Y. Bramouille and R. Kranton (2007), Public goods in networks.
- S. Goyal and J.L Moraga (2001), R&D Networks.
- B. Golub and M.O. Jackson (2008), Naive learning and wisdom of crowds. Stanford University.
- P. De Marzo, D. Vayanos, J. Zweibel (2003), Persuasion bias....